The challenge of teaching children mathematics through meaningful problem solving

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KEY POINTS

- Using a problem-solving approach for teaching mathematics is a true representation of what mathematics is, and what mathematicians do.
- Learning mathematics through problem solving has been shown to be effective for a diverse range of learners.
- Problem solving helps students to develop their understanding at a deeper level than when they memorise or practise given strategies.
- When children learn mathematics through problem solving, it improves their confidence and engenders positive attitudes.
- When teachers work on and solve problems with other teachers, their understandings and dispositions are positively affected.
- A lesson structure using enabling and extending prompts supports implementation in the classroom.

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Problem solving is at the heart of mathematics, and is known to support learning for a diverse range of children. However, challenges can be encountered when teaching mathematics through problem solving. This article provides insights from research literature into (a) the nature of the challenges, and (b) some ways that problem solving can be implemented in the primary classroom. We draw upon some of our own action-research experiences, and those of colleagues, engaged in exploring and learning about teaching mathematics using a problem-solving approach.

Introduction

All mathematics achievement objectives at every level of the New Zealand curriculum refer to mathematics being taught by solving problems (Ministry of Education, 2007). Research literature points to many benefits for learners and teachers, including catering for diversity and supporting children to deeply understand mathematical ideas (Holton et al., 1996; Schoenfeld, 2013; Sullivan et al., 2016). As we have realised ourselves, however, it is one thing to be aware of the benefits of a problem-solving approach, but it is quite another to implement it in our classrooms. To do this it is helpful to know something about the nature of mathematical problems and the problem-solving process, challenges we are likely to encounter, and effective implementation strategies. We explore these in turn in this article, drawing on research literature, our own action-research experiences, and those of colleagues.

The nature of mathematical problems

What is a problem? It is easy to gain a false impression of what constitutes a mathematical problem. For example, it is common to find in commercial mathematics resources (including digital resources) for children the following kinds of questions or activities:

> *Do these problems.*
>
> *6 + 4 =*
>
> *3 + 8 =*
>
> *(and so on)*

These are not true mathematical problems. Rather, they are number sentences. Sometimes contexts are added. For instance:

> *Imagine there are 49 birds sitting in the tree. Another 4 birds come along. How many birds are in the tree now?*

For most learners, this is not a mathematical problem either because the solution method is fairly obvious, may already have been demonstrated, and involves a single step.

***True problems*:** There is no universally accepted definition of what makes a problem, but it is generally thought that in a genuine mathematical problem, the solution method is neither given nor apparent, and the solver does not know how to arrive at an answer (English & Gainsburg, 2016). There may also be one or several solutions. For instance, for young children, the following may be a problem: *The 7 dwarfs ran away from Snow-White and hid in the lounge, some behind the curtain and some behind the sofa. How many dwarfs might have been behind the curtain, and how many behind the sofa?*

Mathematical problem solving involves children building their understanding by developing their own repertoire of strategies to solve problems. In the process, they are likely to experience periods of uncertainty and struggle, and to require support to persist. Problem solving can be time-consuming: a true problem requires time for students to explore, to feel frustration, and to discuss possible strategies. Research findings (Boaler, 2016; Schoenfeld, 2013) indicate clearly, however, that teaching children mathematics through problem solving is time well spent and results in gains for learning and motivation.

A recent problem given to a group of children in my (Hilary's) class illustrates some of these points. It also indicates that what is a problem for one child may not necessarily be a problem for others. The problem involved them investigating the areas of rectangles with the same perimeter.[1] While some children in the group could see immediately that area would not always be the same and could explain this clearly (and was therefore not a "problem" for these children), other children needed to "grapple" with the idea, spend time drawing diagrams, and visualising this. One girl started with a definite opinion that the area would always be the same, but became intrigued by proving

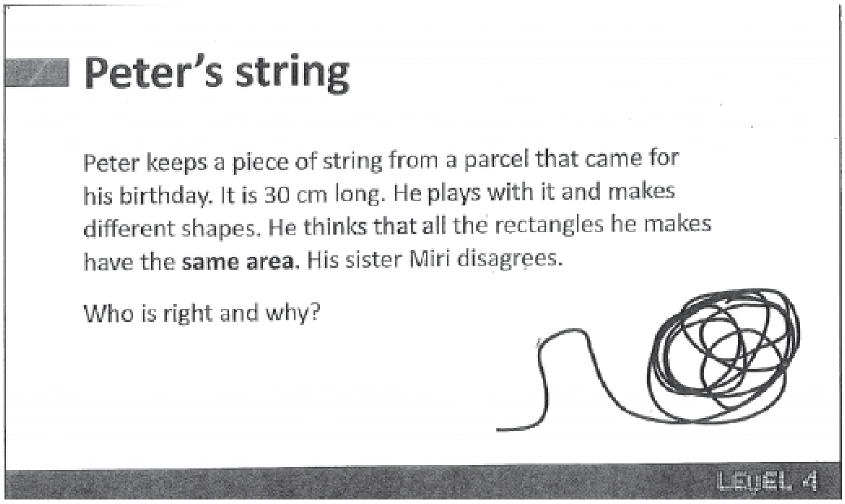


FIGURE 1. PETER'S STRING
(https://nzmaths.co.nz/resource/peter-s-string)

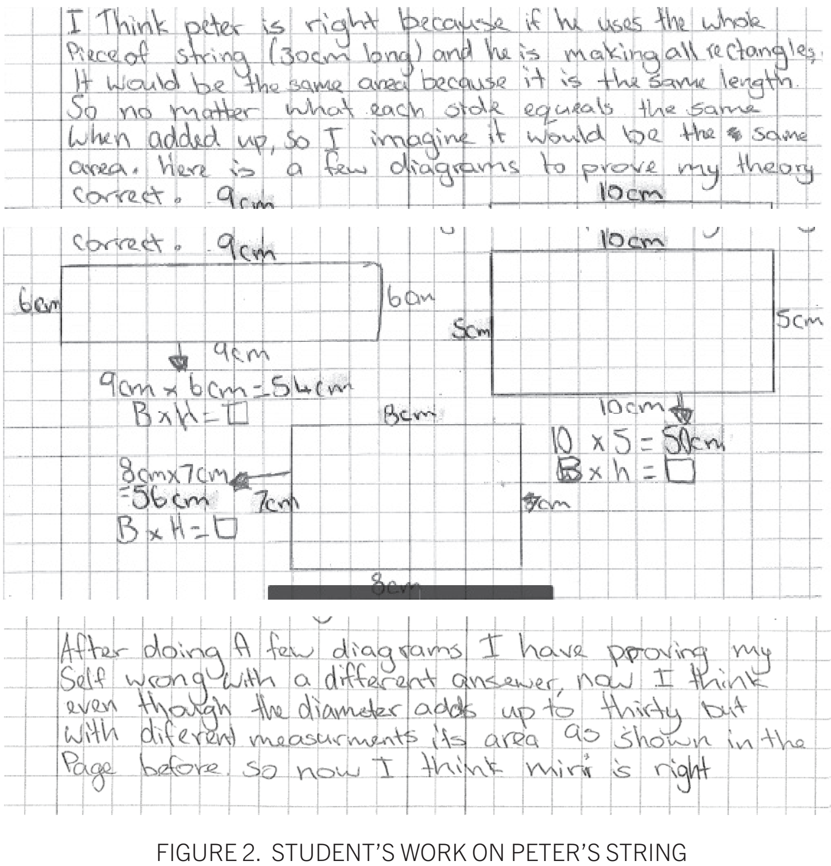


FIGURE 2. STUDENT'S WORK ON PETER'S STRING
(the student shows some confusion about the terms "diameter" and "perimeter")

her initial ideas was incorrect.

***Range of problems*:** Mathematical problems that are meaningful for children can range from those that involve pure number or pure mathematics (*I wonder if there are any patterns in the 9x table?*); through to those with a plausible community and/or real-life context (see Calder, 2013). For example: *The local Tile Company is wondering if we could design some new tiles for them—ones that tessellate.*

Challenges teachers may encounter

As teachers interested in implementing a problem-solving approach with our children we are almost sure to need to address a number of challenges—although it certainly helps if we understand the importance of, and have previously used, an inquiry approach. These challenges, some of which we encountered ourselves, may include the following.

***Compartmentalism*:** Although we may generally value a constructivist approach to children's learning, it is possible that our prior experiences as mathematics learners ourselves have predisposed us to view mathematics teaching and learning as a traditional transmission process (Beswick, 2005). This is likely to have involved demonstration and practice of correct procedures, and for problem solving to have been used only with higher achieving students or as an extra to the everyday mathematics programme (Anderson et al., 2005).

***Pressure of conflicting parent and learner beliefs*:** Child and parent beliefs around the nature of mathematics, and mathematics teaching, can have an influence over classroom practice (Bailey, 2017):

- Many parents also experienced traditional methods of "drill and practise" or "chalk and talk". Consequently, parents and children may well expect that good mathematics teaching involves memorisation of facts and large numbers of exercises completed in exercise books.
- Children may also have come to believe that getting answers correct is sufficient and that there is no need to understand mathematics. Schoenfeld (2013) reported that children whose only experience of mathematics was of traditional exercises that could be solved quickly developed a belief that all problems should be solved

in 5 minutes or less. He found these students ceased working on problems that they might have been able to solve had they persevered.
- Further, Grootenboer and Jorgensen (2009, p. 257) note there is a perception held by the public that mathematics is an individual and "lonely enterprise" rather than the actuality of mathematicians' highly collaborative practice.

Research has reported that it is not unusual that traditional beliefs about mathematics and what constitutes good mathematics teaching can lead to resistance from children and parents when alternative teaching approaches are introduced by teachers. (Within a current project, we are hosting workshops to see if we can support parents' understandings about teaching children mathematics through problem solving.)

***Anxiety and avoidance*:** Mathematics seems to cause anxiety among many people—including preservice teachers who may hold negative attitudes towards mathematics (Young-Loveridge et al., 2012). Researchers such as Boaler (2009) point to a connection between the way mathematics has been traditionally taught—with right/wrong answers, a focus on reproduction of taught methods, and an emphasis on speed of recall and in tests—as being a primary cause of this anxiety. Children often tend to cope with this anxiety by avoiding mathematical struggle or even by avoiding mathematics completely. This is clearly a serious issue when "struggle" and "not knowing" are normal parts of mathematical problem solving. It is pertinent to consider that mathematicians spend their entire careers working on problems they don't know how to solve—at least initially.

***Narrow assessment effects*:** Challenges imposed by "school time structures" can be exacerbated when teachers and children are expected to constantly assess learning against small goals or skills-based learning intentions. These "bite-sized pieces" of mathematics that are quick and easy to test can lead to mathematics becoming fragmented. This makes it difficult for teachers to use problem solving where end-goals are less definite or predictable, and may well differ from child to child.

When a school curriculum mandates skills-based mathematics, with an emphasis on lesson-by-lesson assessment, a larger view of mathematics is unlikely to be realised. In Boaler's view, "when teachers are given lists of content to teach, they see a subject that has been stripped down to its bare parts like a dismantled bike" (2016, p. 31). Under such circumstances, using a problem-solving approach is difficult, and it is thus understandable that problem solving comes to be seen as an "extra" to be undertaken when and if "coverage" of more easily assessable skills is completed.

***Relevant resource availability*:** Several issues have emerged from the research literature (e.g., Anderson, 2005; Schoenfeld, 2016) regarding resources to support problem solving in classrooms:

- Some teachers have experienced difficulty locating suitable problem-solving resources. Some textbooks, for instance, have been found to position problems as either occasional fun activities, or tasks at the end after the "real" mathematics has been finished.
- There are, nevertheless, an increasing number of text and digital resources becoming available (e.g., see Allmond et al., 2010; nzmaths.co.nz/problem-solving; Sullivan, 2018; youcubed.org).
- Availability on its own is not sufficient for many teachers to change their practice. Teachers also appreciate problem-solving resources being demonstrated (Anderson, 2005), and working alongside colleagues who are using the approach.

***Possible influence of colleagues*:** Colleagues who have strongly held beliefs around the value of traditional mathematics teaching can (perhaps inadvertently) discourage teachers keen to embed a problem-solving approach within their practice. This can occur because problem solving requires such a fundamental shift from traditional, didactic mathematics teaching that it can seem like taking a huge, uncomfortable leap of faith into the unknown (Mamolo & Pinto, 2015). The challenges are such that reversion to older, "safer" and more familiar models of teaching is very tempting. Strategies for overcoming the challenges are explored in the next section.

Effective implementation strategies

Despite the challenges, we have found that it is possible to learn to teach children mathematics using a problem-solving approach—complex, but possible. Our own learning has been helped considerably by trying a number of the following strategies that we identified in the research literature.

***Professional development and collaboration*:** Much has been written over the years about the need for professional development and support for teachers implementing problem solving (e.g., Chirinda & Barmby, 2017; Holton et al., 1996). Teacher collaboration and networking are also regarded as essential when reforming practice (Wilson & Cooney, 2002). Internationally, some programmes have been developed where teachers receive formal support and opportunities to network. A recent Australian-based programme—reSolve (maths by inquiry)—promotes the teaching of mathematics as a creative, imaginative endeavour with inquiry and mathematical problem solving at its core. The essence of this programme, which includes both digital resources[2] and face-to-face support for teachers, is the use of volunteer teachers to network and support clusters of schools by facilitating professional learning in accordance with their protocols of teaching mathematics by inquiry.[3]

While New Zealand does not currently have a national professional development programme centred on mathematical problem solving, various providers such as Te Whai Toi Tangata, The Institute of Professional Learning at The University of Waikato[4] have been organising workshops and speakers supporting teachers' learning about problem solving. Jo Boaler from Stanford University and co-developer of the youcubed.org website[5] held a workshop in Hamilton last year hosting 500 teachers from all over the country. Similarly, the Auckland and Waikato Mathematics Associations have hosted speakers such as Anthony Harradine[6] who also supports teachers in their learning about teaching children mathematics via problem solving. The Developing Mathematical Inquiry Communities (DMIC) programme, developed by Dr Bobbie Hunter and colleagues here in New Zealand, is another model with a focus on using challenging problems in mathematical inquiry communities. DMIC links the use of challenging problems to culturally responsive teaching strategies to achieve equitable outcomes for all learners.

***Teachers' first-hand experience with mathematics problem solving*:** Research literature suggests that when teachers work on a range of problems themselves, learning about and internalising processes and problem-solving strategies, they are in a better position to help their children do likewise (e.g., Chirinda & Barmby, 2017; Sullivan et al., 2016). This can extend to groups of teachers (professional learning communities) meeting to collaboratively solve mathematical problems that may then be used in their classrooms. One teacher in a 2011 study noted that a consequence of participating in a professional learning community was feeling validated to use problem solving in her classroom (particularly important when working in a more traditional environment), while others appreciated their own growth in mathematical understandings (Fernandes et al., 2011). Such professional learning communities have been shown to be successful for building teachers' mathematical understandings and dispositions with respect to mathematics problem solving.

We are part of a professional learning research community of four teachers. Our process is to (once every two terms) position ourselves as learners by engaging in several mathematical problems. We then spend time linking these problems to *The New Zealand Curriculum* (Ministry of Education, 2007) (*NZC*) (achievement objectives and key competencies), and writing a detailed unit plan. This is then used over 3 weeks in a Years 5–8 syndicate (*NZC* levels 2–5) operating as an innovative learning environment.

The teachers in this group have valued the time taken to engage with the problems themselves and think about implications for their shared teaching environment. Pearl (all names are pseudonyms) said: "It was good to establish an understanding of what problem solving is (and looks like)", and "I think a big challenge I will have with teaching problem solving is stepping back and not saving students. I know that I tend to jump in too early and need to be very mindful of this when we start teaching lessons." Teresa learnt: "Frustration is okay …" and realised the need to "support and encourage students in the opportunity to think, explore, reason and justify". Like Pearl, she noted the importance of refraining "from jumping in and helping to the point of doing". Emma's thinking included ponderings about "organisation (with respect to the innovative learning environment)—how do we group students and allocate time for teaching via problem solving?".

All of the teachers have noted and appreciated the collaborative, detailed unit planning, and also commented on the value of collaborative reflection. Teresa cited the creation of the plan as very valuable, explaining:

> Having [a slightly adapted, detailed unit plan] at our fingertips and having gone through what may pop up during the lesson was very valuable … [as was] sharing the highs and lows at the end of each lesson and how the next session will look.

Pearl wrote:

> I found the detailed planning done as a team enabled us to feel confident with what we were doing and how to best support the students. It was also good to discuss with my colleagues how things had gone that day and what we would need to adjust/alter before the next session. This would have been harder to do in a single cell without others to bounce ideas off and talk to.

These experiences support what we've read in the research literature, both about the value of positioning ourselves as learners and about the benefits of networking and supporting one another.

***Problem-solving tasks*:** Research has noted that teachers can find it difficult to locate resources that suit a diverse range of learners, including those with poor literacy skills. In her text, *Mathematical Mindsets* (2016), Jo Boaler discusses strategies for changing traditional exercises/tasks into rich tasks accessible for all children. She maintains that it is imperative for children's learning that tasks are open-ended. Boaler's suggestions include:

- "opening" a task to allow for multiple methods, pathways and representations
- changing traditional questions (such as, *What's the area of a 12 by 4 rectangle?*) to an inquiry task (such as, *How many rectangles can you find with an area of 24 square units?*)
- posing the problem before teaching more formal methods
- adding a visual component

- adding the requirement for students to convince and reason.

This last suggestion encompasses the important notion of *justification* which, in practice, means that children not only have to find solutions to problems but also convince others that they are valid solutions.

Adapting tasks as suggested and exemplified in Boaler (2016), and using tasks found on websites such as *nrich*[7]; reSolve (maths by inquiry)[8], and our New Zealand maths website[9] can provide a range of problems for teachers to use with diverse learners. A major benefit for teachers participating in the previously mentioned professional learning communities is to share ideas. Such groups can provide teachers with a stream of worthwhile problems that can be adapted to various classroom scenarios.

Pedagogical strategies to support problem-solving approaches

***Lesson structure*:** Another suggestion to support problem solving in primary mathematics classrooms focuses on a particular lesson structure, including the use of enabling and extending prompts. This comes from the work of Peter Sullivan and colleagues (2016), and is very similar to the structure used in lessons in the DMIC programme (Hunter et al., 2018). Sullivan and colleagues advocate a four-part structure for problem-solving lessons:

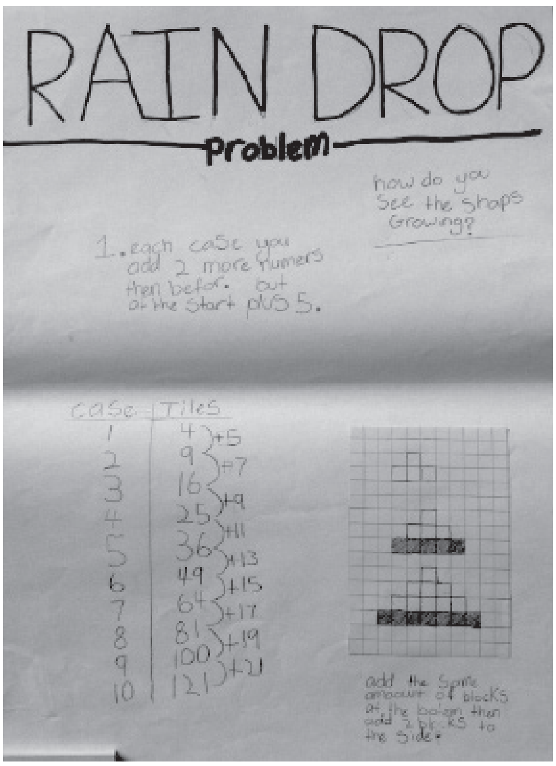
- a first phase where rich/challenging tasks are "launched" and there is an imperative to maintain the "cognitive demand" of the task (i.e., children are not taught what to do)
- an "explore" phase where children work individually or in small groups on the problem
- a "summary" phase where student activity on a problem/task is reviewed as a whole class (and the teacher deliberately sequences student feedback to move from simple to more complex solutions)
- a "consolidation" phase where additional experiences are posed to consolidate the learning activated by the initial task.

We have found it takes time to feel confident with all aspects of this lesson structure. For example, the teachers involved in our current study have found it challenging to maintain the "cognitive demand" required of the launch phase. Teresa mentioned at the end of the first 3-week unit that: "I am still not confident in the way problems were launched as I still question, did we give too much away?" Similarly, Pearl commented: "Launching clearly without giving away possible solutions" was a challenge. These comments are consistent with research findings from Australia (Cheeseman et al., 2016), and is an aspect we are continuing to focus on during the second cycle of our action research.

A key aspect of the "explore" phase is the teacher planning and providing children with enabling prompts and extending prompts, as needed. *Enabling prompts* involve "reducing the number of steps, simplifying the complexity of the numbers, and varying the forms of representation for those students who cannot proceed with the task" (Sullivan et al., 2015, p. 44). It is important to note that these prompts are offered with the explicit intention that the children subsequently return to work on the initial task. *Extending prompts* are offered to students who "complete the original task quickly which ideally elicit abstraction and generalisation of the solutions" (Sullivan et al., 2015, p. 44). Such prompts can occur to teachers when they try the problems themselves first, and were planned for within the unit plan. All teachers valued these prompts and noted this supported them to cater for the range (*NZC* Levels 2–5) of learners in this syndicate. Pearl commented that she "was able to support the students by using enabling & extending prompts". Teresa also appreciated the prompts, explaining: "I found the prompts an important tool, in my hand to refer to, to assist students … to think what they could do next without giving it away."

One example of an enabling prompt included supporting students to focus on how they visualised the pattern growing for the first few cases in "the raindrop task" where they were asked to find out how many squares would be needed for various cases (e.g., Case 10) (see Figure 3).

FIGURE 3. RAINDROP TASK FROM WWW.YOUCUBED.ORG (BOALER, N.D.)



Examples of extending prompts included: *How would you find the number of blocks needed for the 100th pattern? How would you find the number of blocks needed for pattern "n"?* An example of one student's work showing his developing thinking as he grapples with this task can be seen in Figure 4.

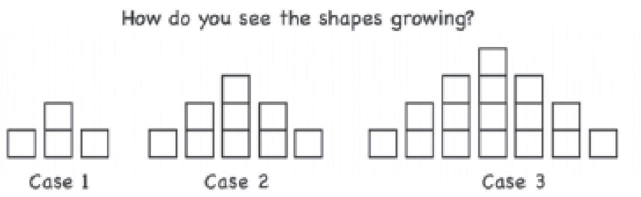


FIGURE 4. STUDENT'S WORK ON THE RAINDROP TASK

***Classroom culture*:** A problem-solving approach in mathematics will work only if there is a respectful classroom climate. It can take up to 6 months of consistent and persistent endeavour to establish this necessary culture.

Boaler (2016) suggests that children who are being introduced to problem solving for the first time may initially be unsure of teacher expectations, and it will take time and conscious effort to effectively alter the learning environment in order for children to feel empowered and willing to take risks. This includes the fostering of an accepting environment where children feel able to make mistakes and ask questions, as well as focus on depth rather than speed (Boaler, 2016).

In terms of children interacting with others, Anthony and Walshaw (2007, p. 72) report that it is, "a major challenge to make classroom discourse an integral part of an overall strategy of teaching and learning"—discourse being a vital aspect of collaborative problem solving.

Chapin and O'Connor (2007) refer to five "talk-moves" that teachers can use to make discussions productive. These include:

- ***revoicing***, where teachers and/or children restate what a previous speaker has said, followed by asking whether the statement is correct (e.g., So you're saying …, is that right?)
- ***repeating***, where a teacher asks a child to repeat what another child has said
- ***reasoning***, where a teacher asks a child, "Do you agree or disagree, and why?"
- ***adding on***, where a teacher asks, "Would anyone like to add to what has already been shared?"
- the well-known ***wait time*** where children are given time to **think** rather than expected to provide quick recall.

We would add a sixth "talk-move" to the list, **enquiring**. We cannot assume that we know what is happening in the minds of our children. It is therefore critical that, rather than checking to see whether they have the so-called right answer, we enquire what our children are actually thinking (e.g., So, what does this part mean? Can you tell us how you worked that out?). In our experience, finding out what is really happening in the heads of our children is one of the delights of teaching.

Another familiar teaching strategy is "*think, pair, and share*" where all children have an opportunity to share their thinking with a "maths buddy".

***Time and school support*:** Since implementation of a problem-solving approach involves a fundamental shift in mathematics learning and teaching in many classrooms, "time" is needed, especially for:

- teacher professional development
- teacher planning
- children to adapt to new expectations.

As indicated previously, this is time well spent because children's learning and attitudes towards mathematics have been found to be enhanced by engaging in problem solving (Boaler, 2016; Schoenfeld, 2013). Research shows when children taught through a problem-solving approach are assessed on mathematics skills, they perform at a similar level to those taught with traditional curricula (Schoenfeld, 2007). However, when tested on conceptual understanding of mathematical ideas and problem solving, they significantly outperform children who have studied more traditional curricula.

Some concluding reflections

We would like to finish by raising a few more issues that need consideration.

Which comes first—skills or problem? It seems logical from an adult point of view to help children first develop the capabilities needed, and then allow them to use these to solve problems. Logical yes, but experience shows that it is much better to give the children the problem first (e.g., I wonder if you could make a model of a geodesic-domed glasshouse as a pattern for my friend who would like to build one?), and they will quickly discover that they need to learn various skills to deal with this problem. A colleague found that the children who tackled this problem learnt the necessary measuring and geometric skills in a fraction of the time it normally took children their age.

***The place of problem solving*:** Is problem solving to be used as an "extra" when time and curricular pressures permit, as enrichment for the most able children, or can it be the means through which all children can learn in mathematics? From our experiences to date, the latter is clearly feasible and desirable.

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Notes

1. See https://nzmaths.co.nz/resource/peters-string
2. Teaching resources and professional development modules: see https://www.resolve.edu.au/teaching-resources
3. See https://www.resolve.edu.au/protocol
4. See https://www.waikato.ac.nz/professionallearning/
5. See https://www.youcubed.org/
6. See https://acems.org.au/mathscraft
7. University of Cambridge, 2017, see https://nrich.maths.org/10334
8. See https://www.youcubed.org/; https://www.resolve.edu.au/teaching-resources
9. See https://nzmaths.co.nz/problem-solving

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