Using PAT: Mathematics to simulate student progress through the National Standards

Elliot Lawes

NZCER
Rangahau Mātauranga o Aotearoa

Contents

Tables

Figures

Summary

The National Standards in Mathematics require teachers to make an Overall Teacher Judgement about each student's achievement. Each student may be judged to be Well Below, Below, At or Above the National Standard for that student's year level. As each student progresses through the year levels, he or she accumulates a sequence of reported achievement categories. This sequence describes that student's progress through the National Standards in Mathematics. For instance, a student who had completed 4 years at school might have a sequence that went Below, At, At, At.

What might we expect the pattern of student progress through the National Standards in Mathematics to look like? This paper uses a robust simulation of student achievement in the National Standards in Mathematics to suggest that:

- approximately one-quarter of students would be judged Well Below or Below *all* of the Year 4 to Year 8 standards as they progress through school
- approximately one-quarter of students would be judged At or Above *all* of the Year 4 to Year 8 standards as they progress through school
- the remaining half of students would be relatively evenly distributed over all of the remaining possible sequences of overall teacher judgement reporting categories.

The simulation in this paper uses PAT: Mathematics assessment data[1] and takes advantage of a study that relates student achievement in PAT: Mathematics with that in the National Standards in Mathematics (Lawes & Darr, 2014).

---

[1] The Progressive Achievement Test (PAT): Mathematics assessment is used by teachers and schools throughout New Zealand. It is psychometrically robust and is explicitly linked to *The New Zealand Curriculum* (Darr, Neill, Stephanou, & Ferral, 2009). It has been developed and is maintained by the New Zealand Council for Educational Research (NZCER), has stood the test of time and is widely trusted.

1. Overview

The National Standards in Reading, Writing and Mathematics are intended to provide a clear framework against which student achievement and progress in reading, writing and mathematics can be assessed using teacher judgement. If they are to succeed at providing this kind of framework, the National Standards need to:

- support consistent teacher judgements of student achievement and progress in each of reading, writing and mathematics
- reflect how students actually achieve and progress in each of reading, writing and mathematics.

If we consider just the first point above, there is evidence to show that teacher judgements of student achievement against the National Standards are not yet consistent and not yet aligned with the expectations of curriculum experts (Lawes, 2016; Ward & Thomas, 2015). The Progress and Consistency Tool (PaCT) (Ministry of Education, 2015b) and the ongoing professional development for teachers seek to address these issues of teacher consistency and expectation. If the PaCT is effective, we might expect to see improvements in teacher consistency and expectation. However, the fact remains that if achievement cannot currently be measured consistently, then progress cannot be measured meaningfully.

But what if teachers were consistent and their expectations of student achievement were aligned with the expectations of curriculum experts? What would we see? This paper puts aside issues of consistency and asks:

> What might we expect the pattern of student progress through the National Standards in Mathematics to look like?

This question is important: its answer provides an exemplar with which we might compare current and future depictions of progress in mathematics.

The simulation in this paper suggests that, for students progressing through the standards from Year 4 to Year 8:

- approximately one-quarter were categorised Well Below or Below *all* of those standards as they progressed through school
- approximately one-quarter were categorised At or Above *all* of those standards as they progressed through school
- approximately half of students would be relatively evenly distributed over all of the remaining possible sequences of overall teacher judgement (OTJ) reporting categories.

The statements above are based on a simulation using assessment data held by the New Zealand Council for Educational Research (NZCER). The assessment in question is the PAT: Mathematics assessment, a sequence of standardised tests developed specifically for use in New Zealand

schools. It is psychometrically robust and was developed to measure achievement in the kind of mathematics described in *The New Zealand Curriculum* (Ministry of Education, 2007).

PAT: Mathematics tests cover the levels of mathematics achievement typically shown by students in Years 3 to 10. The tests are all linked to what is known as the PAT: Mathematics scale. This means that every student who sits a PAT: Mathematics test is assigned a *scale score* based on the number of questions they answered correctly and the test they sat. As students' understanding of mathematics progresses, their scale scores will progress up the PAT: Mathematics scale.

NZCER holds over 450,000 PAT: Mathematics assessment records for over 200,000 students, collected from 2008 to 2013. The Year 4 through to Year 8 students whose assessments are in these data are reasonably representative of all students in these year groups, so we can use these data to tell us about mathematics achievement nationally.

In fact, we can use NZCER's PAT: Mathematics assessment records to simulate mathematics achievement in the National Standards nationally. This is because the recent development of the PaCT provided an explicit relationship between PAT: Mathematics and the National Standards (Lawes & Darr, 2014). And because the PAT: Mathematics data are longitudinal, we can not only use them to simulate *achievement* against the National Standards nationally, we can also use them to simulate *progress* through the National Standards nationally.

The results presented in this paper do not indicate success or failure of the National Standards, teachers or the education system. They are merely what we could expect to see if student achievement in the National Standards followed the same patterns that it exhibits in PAT: Mathematics.

We will begin our journey into understanding what progress through the National Standards in Mathematics might look like by clarifying the paper's main question.

2. Clarifying the main question

This paper addresses the question:

> What might we expect the pattern of student progress through the National Standards in Mathematics to look like?

But what do we mean by the pattern of student progress? When any student completes Year 8 at school, they will have been awarded a sequence of OTJs, one for each National Standard in Mathematics. For an example student, this sequence might look like:

- **At** the after 1 year at school standard
- **At** the after 2 years at school standard
- **Below** the after 3 years at school standard
- **At** the by the end of Year 4 standard
- **At** the by the end of Year 5 standard
- **Below** the by the end of Year 6 standard
- **At** the by the end of Year 7 standard
- **Above** the by the end of Year 8 standard.

This sequence describes the progress of our example student through the National Standards in Mathematics. But what would the collection of these sequences look like for all students completing Year 8 at school in a particular year? As you can imagine, there are many such possible sequences (actually, 65,536). The way this paper addresses the question of what student progress looks like is to estimate the percentages of students associated with each possible sequence of OTJs. For example, perhaps 5 percent of all students could be expected to experience a sequence of OTJs identical to the one described above. The problem with this approach is that 65,536 percentages would have to be estimated and then interpreted—a bewildering prospect.

Instead, the sequences will be grouped by considering only whether each OTJ is either 'Well Below or Below' the standard or is 'At or Above' the standard. These combined OTJ reporting categories will be called 'reporting category groups'. This reduces the workload to estimating only 256 percentages. In addition, the PAT: Mathematics data that this paper's method uses only apply to Years 4 to 8 and so we can only hope to answer our question for those year levels. While this limits the scope of any findings, it does further reduce the workload to estimating 32 percentages.

What kinds of patterns might be revealed by looking at how these 32 percentages describe the distribution of progress through the National Standards in Mathematics? Here are three interesting possibilities:

1. There is a similar percentage of students in each of the sequences of OTJs. This would indicate that progress through the standards was relatively unpredictable given student achievement at any particular standard.

2. There is a high percentage of students in sequences that tend to start Well Below or Below the standard and tend to finish At or Above the standard. These sequences would indicate students making accelerated progress.

3. There is a high percentage of students in sequences that tend to start At or Above the standard and tend to finish Well Below or Below the standard. These sequences would indicate students making decelerated progress.

It is now worth reviewing why we are asking this question. According to the Ministry of Education (2015a), "assessing progress and achievement in relation to the standards is an integral part of teaching and learning across the New Zealand Curriculum". Unfortunately, longitudinal OTJ data do not currently exist at a national level, and there is evidence to show that the national OTJ achievement data that exist are less than consistent (Lawes, 2016; Ward & Thomas, 2015). Therefore it is difficult for teachers, parents and policy makers to talk about what student progress does and should look like. This paper allows us to preview actual national distributions of student progress transformed into progress against the National Standards and to begin to understand and even evaluate those distributions.

Now that we have examined the details of the question this paper addresses and why we might ask that question, we need to consider how it could be answered. As we have hinted above, this paper uses the distributions of student PAT: Mathematics assessment results as a proxy for distributions of teacher OTJs. In order to understand this, we first need to understand a little more about the PAT: Mathematics assessment.

3. The PAT: Mathematics assessment

The PAT: Mathematics assessment is a sequence of standardised tests developed specifically for use in New Zealand schools. The tests are designed to help classroom teachers to understand the achievement and progress of their students in mathematics as it is described in *The New Zealand Curriculum* (Ministry of Education, 2007).

PAT: Mathematics tests cover the levels of mathematics achievement typically shown by students in Years 3 to 10. The tests are all linked to what is known as an *equal interval scale*. Every student who sits a PAT: Mathematics test is assigned a *scale score* based on the number of questions they answered correctly and the test they sat. The fact that the scale is an equal-interval scale means that progress of one unit up the scale indicates the same amount of mathematical progress no matter where on the scale the progress occurs.

As an example, a Year 4 student who scored 50 percent correct on the Year 4 test will earn a scale score of 29.7 scale units, whereas a Year 8 student who also scored 50 percent correct on the much more difficult Year 8 test will earn a scale score of 56.3 scale units. While the percentage correct for the two students is the same, the two students are clearly at different levels of mathematics achievement (because one of the tests was much more difficult than the other). This differing amount of mathematics achievement *is* captured by the PAT: Mathematics scale score.

The scale is known as the PAT: Mathematics scale and its units are called *patm* units. The PAT: Mathematics scale is analogous to a temperature scale, and the *patm* units are analogous to degrees Celsius (ºC), which are units of measurement on a scale of temperature.

The PAT: Mathematics scale allows us to look at patterns in the distributions of student achievement for students in different year levels. For example, the distribution of PAT: Mathematics scale scores of a nationally representative group of Year 8 students on the PAT: Mathematics scale in Term 4 is shown in Figure 1.

Figure 1 plots the distribution of achievement for Year 8 students on the PAT: Mathematics scale using the results from NZCER's assessment data for tests completed in Term 4. The distribution is normal (i.e. bell-shaped), with an average of 58.8 *patm* and a standard deviation (an indicator of variability) of 12.2 *patm*. We can also see that around 68.2 percent of Year 8 students scored between 71.0 *patm* (one standard deviation above the mean) and 46.6 *patm* (one standard deviation below the mean). These numbers are slightly different from the Term 1 distributions reported in Darr et al. (2009): we use Term 4 records from NZCER's assessment data because it is in Term 4 that OTJs are made.

Figure 1   **The distribution of Year 8 students' PAT: Mathematics scores**

PAT: Mathematics scale
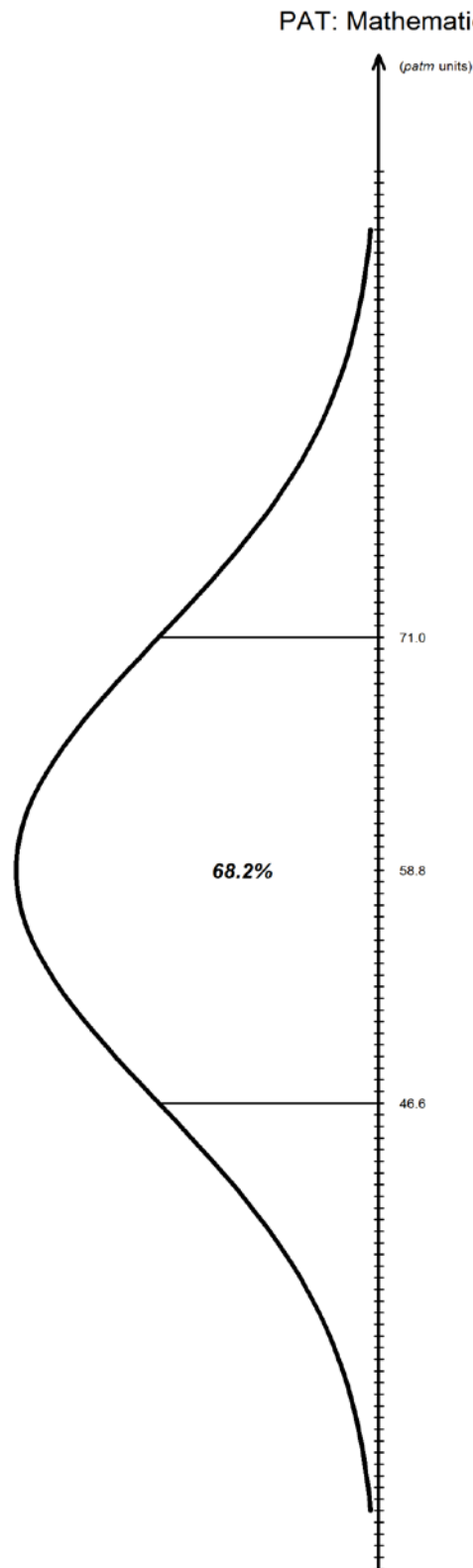
The PAT: Mathematics scale also allows us to look at how students make progress in Mathematics. Figure 2 depicts a situation where a student is assessed in Term 1 at the beginning of Year 6 (scoring 36 *patm*), in Term 4 at the end of Year 7 (scoring 63 *patm*) and at the end of Year 8 (scoring 72 *patm*). We can see their scale score for each assessment as well as a

confidence interval around that score indicating the precision of measurement for each assessment. Note that in Figure 2, data from term 1 and term 4 is used for illustrative purposes whereas in the analysis in the rest of the paper, only data from term 4 is used.

It is this type of longitudinal data about groups of students, together with a link between the PAT: Mathematics scale and the OTJ reporting categories of the National Standards in Mathematics, that allows us to study student progress through the standards.

Figure 2   **The progress of a student shown on the PAT: Mathematics scale**
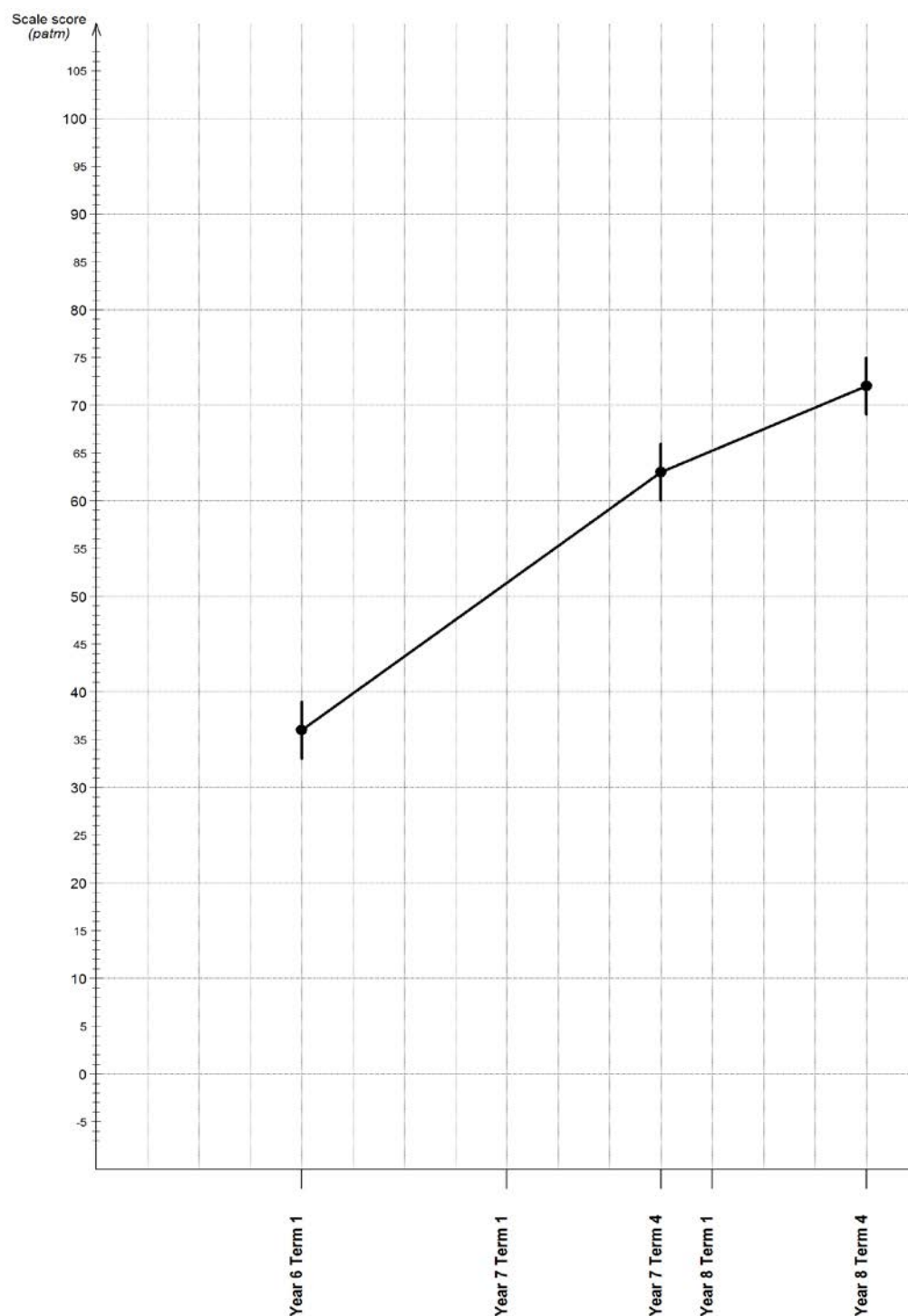
4. OTJs on the PAT: Mathematics scale

So how do we link the PAT: Mathematics scale and the OTJ reporting categories of the National Standards in Mathematics?

The main mechanism for this translation comes from the development work associated with the PaCT — a resource intended to improve the consistency of OTJs (Ministry of Education, 2015b). As part of the development of the PaCT, a group of curriculum experts in mathematics was convened to determine the points on the PaCT scale that represent the minimum levels of performance required to be judged at each National Standards reporting category. These points are known as *cut scores*. Also during the development of the PaCT, a relationship was determined between the PaCT mathematics scale and the PAT: Mathematics scale (Lawes & Darr, 2014). This relationship can be used to transfer the cut scores on the PaCT mathematics scale to the PAT: Mathematics scale. These cut scores are shown in Table 1.

Table 1 **Standard-setting cut scores determined by curriculum experts (from the PaCT)**

PAT: Mathematics cut score for year-level standards (*patm* units)						
At after 3	**At 4**	**At 5**	**At 6**	**At 7**	**At 8**	**Above 8**
32.8	38.3	43.7	49.7	54.8	61.6	68.3

How do we read Table 1? As an example, Table 1 says that in the absence of any other information about the mathematics performance of a student other than a recent PAT: Mathematics scale score of 56.3 *patm*, our best estimate of that student's OTJ would be that they were at the end of Year 7 standard in mathematics.

It is worth noting that, while we can mathematically translate the cut scores from the PaCT tool to the PAT: Mathematics assessment scale, these two assessments have different intentions and we should be careful how we interpret the cut scores in Table 1. In particular, teachers consider a broad range of evidence when they use the PaCT tool to make an OTJ, whereas students answer around 40 questions when they generate a scale score using the PAT: Mathematics assessment. These 40 questions are indicative of the breadth of the curriculum, but they are still only 40 questions. So, for a given student, the cut scores in Table 1 would only provide a very rough guide to a teacher on how to assign an OTJ to that student. To assign an OTJ properly to that student, the teacher would have to consider a broader range of evidence.

However, over a population of students, the distribution of OTJs determined by teachers using the PaCT should robustly simulate the distribution obtained by transforming PAT: Mathematics scores into OTJs using Table 1.

Using Table 1 and NZCER's PAT: Mathematics assessment records, we can also look at the progress of students through the National Standards. The mechanism of translation works as we discussed above. For example, consider a student scoring 35 *patm* units on a PAT assessment near the end of Year 4 and 45 *patm* units on a PAT assessment near the end of Year 5. Because their first assessment score of 35 *patm* units is less than the At 4 cut score of 38.3, the student would

most likely be described as being Well Below or Below the Year 4 standard. Similarly, because their second assessment score of 45 *patm* units is greater than the At 5 cut score of 43.7, the student would most likely be described 1 year later as being At or Above the Year 5 standard.

As in our discussion of achievement above, using only PAT: Mathematics scores and Table 1 to describe the progress of any individual student through the National Standards would have dubious validity. OTJs, whether in the context of achievement or progress, are intended to be informed by a broad range of evidence. Again, however, over a population of students, the distribution of progress in OTJs determined by teachers should robustly simulate the distribution of progress obtained by transforming PAT: Mathematics scores into OTJs using Table 1.

And this is the core of this paper's method. It simply applies Table 1 to distributions of progress derived from the PAT: Mathematics data from NZCER's assessment records. The next section shows how this plays out for data focused on the progress that occurs between Years 4 and 5.

5. The Years 4 and 5 standards

To produce a view of progress through the National Standards in Mathematics from Year 4 to Year 5, we need to apply the OTJs specified in Table 1 to a large, representative sample of students with a PAT: Mathematics assessment record from the end of Years 4 and 5. When we do this using NZCER's PAT: Mathematics assessment data, we get Table 2.

Table 2 is quite complicated, so we will explore it fully. The top-left, top-centre, centre-left and centre-centre numerical entries (all unshaded) make up the most important part of the table. The percentages in Table 2 describe students who were assessed at the end of Year 4 and again at the end of Year 5.

- The top-left numerical entry (38.54 percent) means that around 39 percent of these students were categorised Well Below or Below *both* the Year 4 standard and the Year 5 standard.
- The top-centre numerical entry (11.07 percent) means that around 11 percent of these students were categorised Well Below or Below the Year 4 standard *and* At or Above the Year 5 standard.
- The centre-left numerical entry (6.24 percent) means that around 6 percent of these students were categorised At or Above the Year 4 standard *and* Well Below or Below the Year 5 standard.
- The centre-centre numerical entry (44.15 percent) means that around 44 percent of these students were categorised At or Above *both* the Year 4 standard and the Year 5 standard.

The remaining entries of the table are various sums of the above four entries.

- The bottom-right numerical entry (100 percent) is the overall sum of the four entries discussed above. This represents all of the students who were assessed at the end of Year 4 and again at the end of Year 5. This entry is the sum of the two entries in Table 2 that sit to the left of it. It is also the sum of the two entries in Table 2 that sit above it.
- The top-right numerical entry (49.60 percent) means that around 50 percent of these students were categorised Well Below or Below the Year 4 standard (regardless of their categorisation against the Year 5 standard). This entry is the sum of the two entries in Table 2 that sit to the left of it.
- The centre-right numerical entry (50.40 percent) means that around 50 percent of these students were categorised At or Above the Year 4 standard (regardless of their categorisation against the Year 5 standard). This entry is the sum of the two entries in Table 2 that sit to the left of it.
- The bottom-left numerical entry (44.78 percent) means that around 45 percent of these students were categorised Well Below or Below the Year 5 standard (regardless of their categorisation against the Year 4 standard). This entry is the sum of the two entries in Table 2 that sit above it.
- The bottom-centre numerical entry (55.22 percent) means that around 55 percent of these students were categorised At or Above the Year 5 standard (regardless of their categorisation against the Year 4 standard). This entry is the sum of the two entries in Table 2 that sit above it.

Table 2 **Percentages of students in relation to the Year 4 and Year 5 National Standards in Mathematics**

	Well Below or Below the Year 5 standard %	At or Above the Year 5 standard %	Total %
Well Below or Below the Year 4 standard	38.54	11.07	49.60
At or Above the Year 4 standard	6.24	44.15	50.40
Total	44.78	55.22	100.00

By adding the top-left (38.54 percent) and centre-centre (44.15 percent) entries of Table 2, we notice that around 83 percent of students who were assessed at the end of Year 4 and again at the end of Year 5 were likely to have the same OTJ reporting category group for both standards.

Another way of thinking about the distribution of progress from Year 4 to Year 5 is to estimate, for each Year 4 reporting category group, the percentages of students who end up in each Year 5 reporting category group. We can do this by dividing the percentages in the left and centre columns of Table 2 by those in the right-hand column. For example, approximately 77.7 percent of students who were categorised Well Below or Below the Year 4 standard were also categorised Well Below or Below the Year 5 standard. Here we calculated 77.7 percent by dividing 38.54 percent in the top-left of Table 2 by 49.60 percent in the top-right. This analysis is presented in Table 3.

Table 3 **Percentages of students in each Year 5 National Standards reporting category group by Year 4 National Standards reporting category group**

	Well Below or Below the Year 5 standard %	At or Above the Year 5 standard %	Total %
Well Below or Below the Year 4 standard	77.70	22.32	100
At or Above the Year 4 standard	12.38	87.60	100
Total	44.78	55.22	100

Notice from Table 3 that, of the students who were categorised Well Below or Below the Year 4 standard, around 77.7 percent were also categorised Well Below or Below the Year 5 standard, as we calculated above.

We could also think about progress from Year 4 to Year 5 by estimating, for each Year 5 reporting category group, the percentages of students in each Year 4 reporting category group. We could do this by dividing the percentages in the top and centre rows of Table 2 by those in the bottom row. However, I will not go through this in detail.

But Years 4 and 5 were not special here. We could produce tables analogous to Table 2 and Table 3 that describe the transition from: the Year 5 to the Year 6 standards; the Year 6 to the Year 7 standards; and the Year 7 to the Year 8 standards. And if we could do this, could we somehow combine the information in those tables to investigate progress through all of the National Standards from Year 4 to Year 8? Yes we could, but to fully understand the details of how this could be done, we first look at the transition summary statistics from the Year 5 to the Year 6 standards and how we can combine these with the summary statistics for the transition from the Year 4 to the Year 5 standards.

6.   The Years 5 and 6 standards

If the OTJ cut scores specified in Table 1 are applied to a large, representative sample of students with a PAT: Mathematics assessment record from the end of Years 5 and 6, we get Table 4. Table 4 can be read like Table 2. As a reminder, notice that around 9 percent of students who were assessed at the end of Year 5 and again at the end of Year 6 were categorised At or Above the Year 5 standard and Well Below or Below the Year 6 standard. The percentages in Table 4 are quite similar to those in Table 2.

Table 4 **Percentages of students in relation to the Year 5 and Year 6 National Standards in Mathematics**

	Well Below or Below the Year 6 standard %	At or Above the Year 6 standard %	Total %
Well Below or Below the Year 5 standard	39.49	6.00	45.49
At or Above the Year 5 standard	9.22	45.29	54.51
Total	48.71	51.29	100.00

As we did for Table 2, we can estimate, for each Year 5 reporting category group, the percentages of students in each Year 6 reporting category group. Again, we can do this by dividing the percentages in the centre and left columns of Table 4 by those in the right-hand column. For example, approximately 86.81 percent of students who were categorised Well Below or Below the Year 5 standard were also categorised Well Below or Below the Year 6 standard. Here we calculated 86.81 percent by dividing 39.49 percent in the top-left of Table 4 by 45.49 percent in the top-right. The results are shown in Table 5.

Table 5 **Percentages of students in each Year 6 National Standards reporting category group by Year 5 National Standards reporting category group**

	Well Below or Below the Year 6 standard %	At or Above the Year 6 standard %	Total %
Well Below or Below the Year 5 standard	86.81	13.19	100
At or Above the Year 5 standard	16.91	83.09	100
Total	48.71	51.29	100

So how can we combine the Years 4 and 5 information from Section 5 and the Years 5 and 6 information from the current section? As an example of how we might combine this information, we estimate the percentage of students who had the following sequence of reporting category group judgements:

- Well Below or Below the Year 4 standard
- Well Below or Below the Year 5 standard
- At or Above the Year 6 standard.

Table 2 shows us that approximately 49.6 percent of students were categorised Well Below or Below the Year 4 standard. Table 3 shows us that approximately 77.7 percent of those students who were categorised Well Below or Below the Year 4 standard were categorised Well Below or Below the Year 5 standard. Table 5 shows us that approximately 13.19 percent of those students who were categorised Well Below or Below the Year 5 standard were categorised At or Above the Year 6 standard. Multiplying these three percentages gives us an estimate of the percentage of students who would have had the following sequence of judgements: Well Below or Below the Year 4 standard; Well Below or Below the Year 5 standard; and At or Above the Year 6 standard. This results in an estimate of approximately 5.08 percent of students.

If we use a '0' as a shorthand for being categorised Well Below or Below the standard and a '1' as a shorthand for being categorised At or Above the standard, the above sequence of judgements would have the shorthand form: 0;0;1. Table 6 shows all possible sequences through the Year 4, Year 5 and Year 6 standards using this shorthand. As an example of how to read Table 6, we can see that the sequence described above is the second row of numbers in the table.

Notice that the sequences where the three judgements are either all At or Above the standard or all Well Below or Below the standard are exhibited in substantially higher percentages than the other sequences.

Section 7 displays the tables analogous to Table 2 and Table 4 that describe changes in OTJ reporting category groups from Year 6 to Year 7 and from Year 7 to Year 8. Section 8 is focused on extending Table 6 by incorporating the changes in OTJ reporting category groups from Year 6 to Year 7 and from Year 7 to Year 8.

Table 6 **Percentages of students in relation to the Year 4, Year 5 and Year 6 National Standards in Mathematics**

Sequence			Percentage in sequence			
At or Above the Year 4 standard	At or Above the Year 5 standard	At or Above the Year 6 standard	Percentage of students with reporting category group in sequence at Year 4	Percentage of students with reporting category group in sequence at Year 5 given their Year 4 reporting category group	Percentage of students with reporting category group in sequence at Year 6 given their Year 5 reporting category group	Estimated percentage of students
0	0	0	49.60	77.70	86.81	33.46
0	0	1	49.60	77.70	13.19	5.08
0	1	0	49.60	22.32	16.91	1.87
0	1	1	49.60	22.32	83.09	9.20
1	0	0	50.40	12.38	86.81	5.42
1	0	1	50.40	12.38	13.19	0.82
1	1	0	50.40	87.60	16.91	7.47
1	1	1	50.40	87.60	83.09	36.68

7. The Years 4, 5, 6, 7 and 8 standards

This section presents tables that describe all of the changes in OTJ reporting category groups for each year-to-year transition in Years 4, 5, 6, 7 and 8. These tables are mostly included for the sake of completeness, but it is worth noticing how similar they are. The reason for this is largely because at each year level the cut score for being At or Above the standard is quite close to the average PAT: Mathematics score for students in that year level. Put another way, if the relative position of the average PAT: Mathematics score and the year-level cut score varied substantially by year level, then the percentages in Table 7, Table 8, Table 9 and Table 10 would also vary substantially. Note that Table 7 reproduces Table 2 and Table 8 reproduces Table 4.

Table 7 **Percentages of students in relation to the Year 4 and Year 5 National Standards in Mathematics**

	Well Below or Below the Year 5 standard %	At or Above the Year 5 standard %	Total %
Well Below or Below the Year 4 standard	38.54	11.07	49.60
At or Above the Year 4 standard	6.24	44.15	50.40
Total	44.78	55.22	100.00

Table 8 **Percentages of students in relation to the Year 5 and Year 6 National Standards in Mathematics**

	Well Below or Below the Year 6 standard %	At or Above the Year 6 standard %	Total %
Well Below or Below the Year 5 standard	39.49	6.00	45.49
At or Above the Year 5 standard	9.22	45.29	54.51
Total	48.71	51.29	100.00

Table 9 **Percentages of students in relation to the Year 6 and Year 7 National Standards in Mathematics**

	Well Below or Below the Year 7 standard %	At or Above the Year 7 standard %	Total %
Well Below or Below the Year 6 standard	43.97	7.81	51.78
At or Above the Year 6 standard	7.12	41.10	48.22
Total	51.09	48.91	100.00

Table 10 **Percentages of students in relation to the Year 7 and Year 8 National Standards in Mathematics**

	Well Below or Below the Year 8 standard %	At or Above the Year 8 standard %	Total %
Well Below or Below the Year 7 standard	47.57	5.05	52.62
At or Above the Year 7 standard	9.51	37.87	47.38
Total	57.09	42.91	100.00

These Tables are slightly overwhelming seen one after another and might prompt one to ask, "Why not just use the PAT: Mathematics records of students who have been assessed at the ends of all of Years 4, 5, 6, 7 and 8?" The answer is that the group of students who fit this criterion is not representative of all students in Years 4 to 8. Schools choose to use the PAT: Mathematics assessment and choose *when* they use it. Therefore NZCER's PAT: Mathematics assessment records are an administrative data set that reflects school choice rather than what might have been collected in a purposeful study.

While the set of students with data in PAT: Mathematics assessment records who were assessed at the ends of all of Years 4, 5, 6, 7 and 8 are not representative of all students in those years, it has been possible to draw a representative sample of students assessed at the end of both Years 4 and 5; the same goes for Years 5 and 6, Years 6 and 7 and Years 7 and 8. This is why this paper has used a methodology that relies on all of the tables in this section.

8. Conclusion: Progress through the standards

Table 11 (see over) displays all possible sequences of OTJ reporting category groups through the National Standards in Mathematics, together with an estimate of the percentage of students receiving that sequence of OTJs. The percentages are estimated using the method described in Section 6 and the tables in Section 7 and rely on the assumption that we can use PAT: Mathematics data and the relationship between student achievement on the PaCT and student achievement in PAT: Mathematics to robustly simulate the distributions of student OTJs.

How do we read Table 11? As an example, the third line of numbers tells us that under this assumption approximately 1.01 percent of students would be awarded the following sequence of OTJ reporting categories as they progressed through the National Standards in Mathematics:

- Well Below or Below the Year 4 standard
- Well Below or Below the Year 5 standard
- Well Below or Below the Year 6 standard
- At or Above the Year 7 standard
- Well Below or Below the Year 8 standard.

What does Table 11 tell us? If we assume that we can use PAT: Mathematics data and the relationship between student achievement against the National Standards in Mathematics and student achievement in PAT: Mathematics to robustly simulate the distributions of student OTJs, then:

- the mathematics performance of approximately one-quarter of students would be judged Well Below or Below *all* of the Year 4 to Year 8 standards as they progress through school
- the mathematics performance of approximately one-quarter of students would be judged At or Above *all* of the Year 4 to Year 8 standards as they progress through school
- the mathematics performance of the remaining half of the students would be relatively evenly distributed over the remaining 30 possible sequences of OTJ reporting category groups.

A slightly different view of Table 11 is presented by counting the total number of times At or Above the standard appears in a sequence. This is presented in Table 12. Here our simulation shows more clearly that about half of all students would experience a sequence of judgements that are either consistently At or Above or consistently Well Below or Below each year level standard in Years 4 to 8.

Table 11 **Sequences of OTJ reporting category groups through the National Standards in Mathematics**

At or Above the Year 4 standard	At or Above the Year 5 standard	At or Above the Year 6 standard	At or Above the Year 7 standard	At or Above the Year 8 standard	Estimated percentage of students
0	0	0	0	0	25.64
0	0	0	0	1	2.68
0	0	0	1	0	1.01
0	0	1	0	0	0.69
0	1	0	0	0	1.41
1	0	0	0	0	4.10
0	0	0	1	1	4.11
0	0	1	0	1	0.07
0	1	0	0	1	0.15
1	0	0	0	1	0.43
0	0	1	1	0	0.85
0	1	0	1	0	0.06
1	0	0	1	0	0.16
0	1	1	0	0	1.25
1	0	1	0	0	0.11
1	1	0	0	0	5.70
0	0	1	1	1	3.46
0	1	0	1	1	0.23
1	0	0	1	1	0.66
0	1	1	0	1	0.13
1	0	1	0	1	0.01
1	1	0	0	1	0.60
0	1	1	1	0	1.53
1	0	1	1	0	0.14
1	1	0	1	0	0.22
1	1	1	0	0	5.03
1	1	1	1	0	6.19
1	1	1	0	1	0.53
1	1	0	1	1	0.91
1	0	1	1	1	0.55
0	1	1	1	1	6.24
1	1	1	1	1	25.16

Table 12 **The number of times a sequence includes 'At or Above the standard'**

Total times a sequence includes At or Above the standard	Number of sequences with this total	Estimated percentage of students according to curriculum experts
0	1	25.64
1	5	9.90
2	10	12.89
3	10	12.00
4	5	14.42
5	1	25.16

There are other interesting ways to filter Table 11. For example, we could look at the percentages associated with all sequences that start with a judgement of Well Below or Below the Year 4 standard and finish with a judgement of At or Above the Year 8 standard. This would represent those students whose mathematics proficiency was in a better place relative to the National Standards in Year 8 than it was in Year 4.

However, providing a comprehensive picture of progress through the National Standards is best left to a time when we have empirical progress data to inform this picture.

9. Discussion

The value of this paper is that it is predictive. The paper asks, "What might student progress through the National Standards in Mathematics look like when we have the longitudinal student data to tell?" As we've already seen, the paper provides estimates of the percentages of students with various sequences of OTJ reporting groups. If we assume that these percentages are a realistic representation of the distribution of student progress through the National Standards in Mathematics, then we might be prompted to ask:

1.   Is the National Standards system a useful way to describe student progress nationally?
2.   If it is, what kinds of distributions of progress are good enough to reach New Zealand's educational goals?
3.   If we are interested in telling whether or not our distribution of progress is good enough, what kind of indicators can we use?

The first question is complex. It invites us to think about how we would like to see student progress described nationally and how this should relate to how we describe achievement nationally. There are many ways to describe progress, each with its associated costs and benefits. By way of example, this paper shows that describing progress using National Standards reporting categories is combinatorially intensive, even though the progress described is clearly linked to achievement at every year level.

But whatever the method chosen to describe progress (including the method used by the National Standards), it must result in distributions of progress that are both believable and useful. Do we really think that a quarter of all students are consistently failing to meet our expectations and another quarter are consistently at or above those same expectations? And what are we to do with this information? If we were designing policy or an educational intervention to address this *progress* issue, what would we do differently from what we do now when we think mostly about student *achievement*?

The second question asks us what we want progress to look like and how we should evaluate progress against the National Standards. If it was real and not just part of a theoretical exercise, would the distribution of progress described in this paper be acceptable for a country like ours? Perhaps we would like to see fewer students consistently Well Below or Below the standards? Would a fifth of all students be acceptable? And how realistic would a shift from a quarter to a fifth be?

Perhaps it is the particular shape of the distribution in this paper that is not quite good enough. Would we like to see consistently higher percentages of students transition from Well Below or Below the standards to At or Above? What would this mean for the percentages transitioning in the other direction? What would it mean for distributions of achievement?

This leads naturally onto the third question: How do we tell if our distribution of progress is good enough? There are many possible progress indicators that could be derived from Table 11. Two examples are:

- the percentage of students who would be judged Well Below or Below *all* of the standards
- the percentage of students whose progress would begin with a number of Well Below or Below judgements against consecutive standards, followed by a number of At or Above judgements against consecutive standards.

The first indicator captures the proportion of students who make little progress relative to the National Standards. A fuller exploration of progress could refine this indicator to capture only those students who would be judged Well Below *all* of the standards.

The second indicator captures the proportion of students who make accelerated progress relative to the National Standards. This indicator is reflected in Table 11 by those students whose progress sequences are a number of 0s followed by a number of 1s, and describes the progress of approximately 21 percent of students. Again, a fuller exploration of progress could refine this indicator to incorporate sequential progress through each of the National Standards reporting categories. If we were able to monitor how progress changed over time, we would obviously like the first indicator to decrease and the second indicator to increase. However, it is difficult to know how quickly we might expect these indicators to change. But these two indicators are only examples of the kinds of tools that might be useful when describing progress.

Finally, it is worth recalling that the distribution of progress shown in this paper is simulated. Currently, there is no national longitudinal collection of OTJs that we can use to check the questions above. If we hope to use the National Standards system to measure student progress, then we must plan to investigate the questions above using high-quality real data.

References

Darr, C., Neill, A., Stephanou, A., & Ferral, H. (2009). *Progressive Achievement Test: Mathematics, Teacher manual, 2nd edition, Revised.* Wellington: New Zealand Council for Educational Research.

Lawes, E. (2016). *Using PAT: Mathematics to explore the consistency of teacher judgements against the National Standards*. Wellington: New Zealand Council for Educational Research.

Lawes, E., & Darr, C. (2014). *The Progress and Consistency Tool: Mathematics framework technical report.* Wellington: New Zealand Council for Educational Research.

Ministry of Education. (2007). *The New Zealand curriculum: For English-medium teaching and learning in years 1–13*. Wellington: Learning Media.

Ministry of Education. (2015a). *National Standards.* Wellington: Author. Retrieved from http://www.education.govt.nz/ministry-of-education/specific-initiatives/national-standards/

Ministry of Education. (2015b). *The Progress and Consistency Tool (PaCT): Supporting professional judgements in reading, writing, and mathematics.* Wellington: Author. Retrieved from http://pactinfo.education.govt.nz/

Ward, J., & Thomas, G. (2015). *National Standards: School sample monitoring and evaluation project, 2010−2013*. Wellington: Ministry of Education. Retrieved from http://www.educationcounts.govt.nz/publications/series/National_Standards/national-standards-school-sample-monitoring-and-evaluation-project-2010-2013