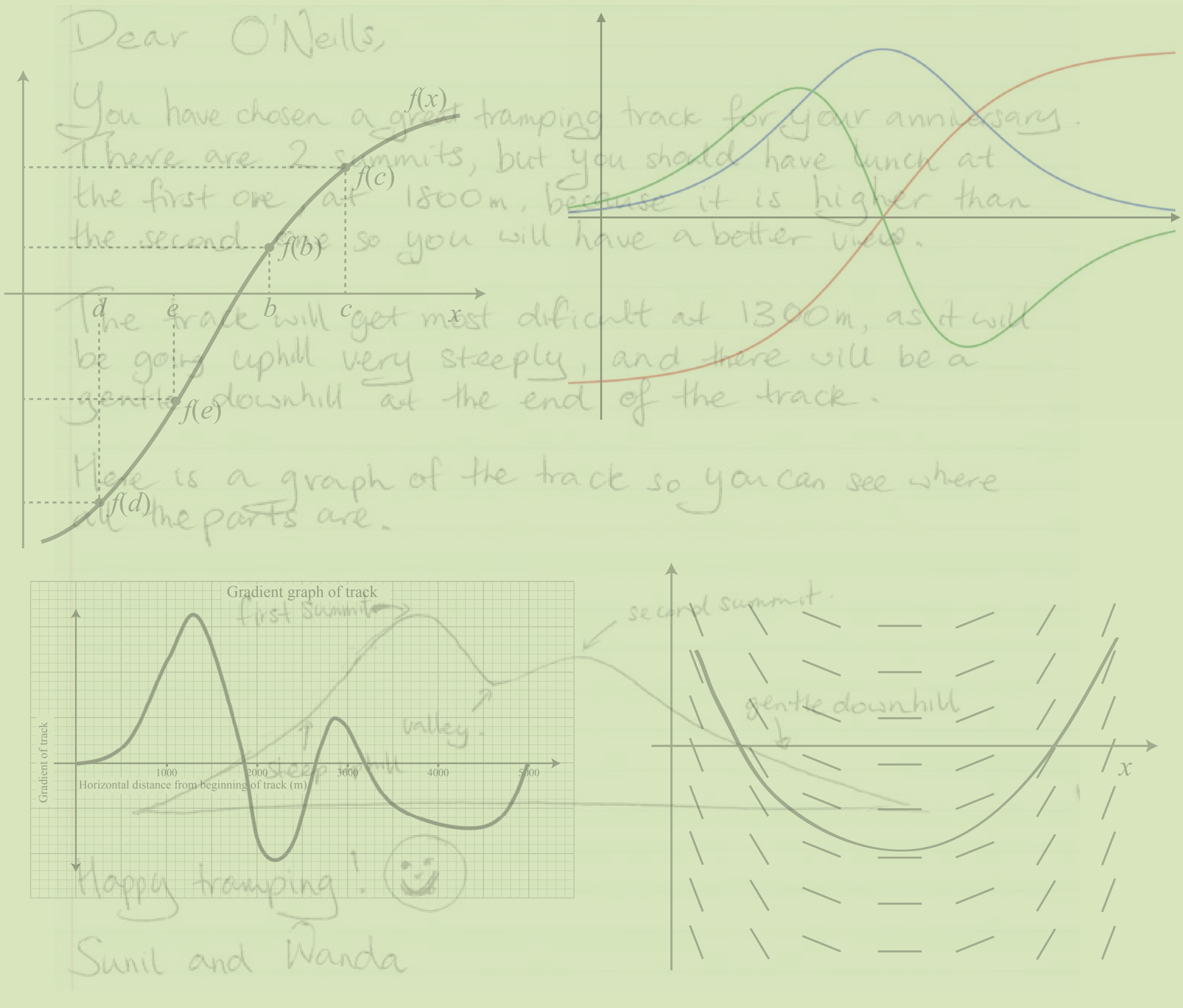
Graphical Antiderivatives

TEACHER MANUAL



By Caroline Yoon, Tommy Dreyfus, Tessa Miskell and Mike Thomas

5 The LEMMA series: Learning Encounters with Meta-Mathematical Activities

NZCER
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United Nations
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Introduction

Warm-up task

Overview: The goal of this warmup is to prepare students for Task 1 (the Tramping problem), by engaging them in the tramping context and mathematical concepts. It takes 15 minutes, and can be done with the whole class either the day before Task 1, or at the beginning of the class session when Task 1 is implemented. The benefit of doing the warmup the day before is that it will give students more time to work on their written letters for Task 1, and may leave some room for student presentations at the end. However, it also works well if the warmup is presented on the same day as Task 1, as long as the warmup is completed within 10 minutes to leave enough time for the Tramping problem.

Watch the first minute of the slip n slide video and answer the following questions.

http://www.youtube.com/watch?v=DPp2HlIMkmU

Note: These questions are designed to engage students in the context of the problem. It is efficient to go through these questions as part of a whole class discussion, rather than having students write their answers individually.

Discuss with your classmates, the following questions:

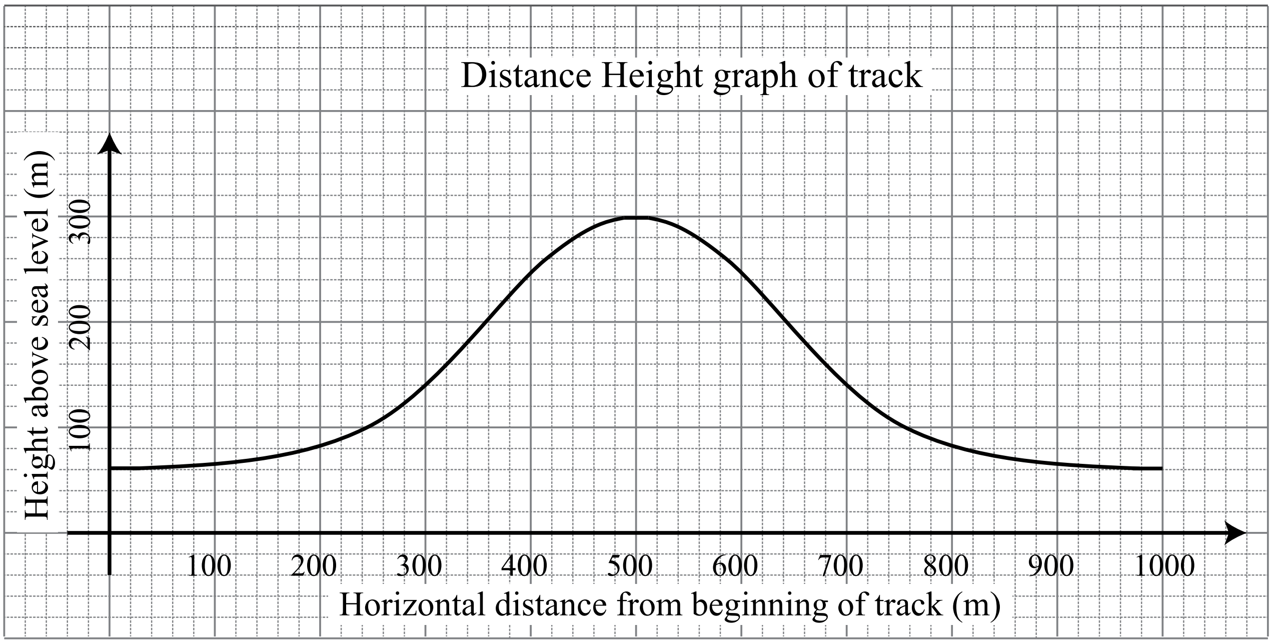
- Use your left hand to show the slope of the slide.
- Why does the slide slope upwards at the end?

Answer: So the person on the slide will be airborne.

- Is there any part of the slide that is flat? If so, where?

Answer: Yes, at the bottom part, just before it goes back up again.

Below is a graph of a tramping track that goes up and down a hill.



1. What is the horizontal length of this track? (Note: this is different from the actual length along the track)

Answer: 1000 m

2. At what horizontal distance is the track the highest?

Answer: 500 m

3. How high is the highest part of the track?

Answer: 300 m

4. Where does the track appear to be steepest uphill?

Answer: about 350 m

We can measure how steep the track is at any point by finding the *gradient of the tangent line* to the graph of the track.

HOW TO: Find the gradient of the tangent line at a point

(a) Draw a tangent line to the graph at a point (x_1, y_1)

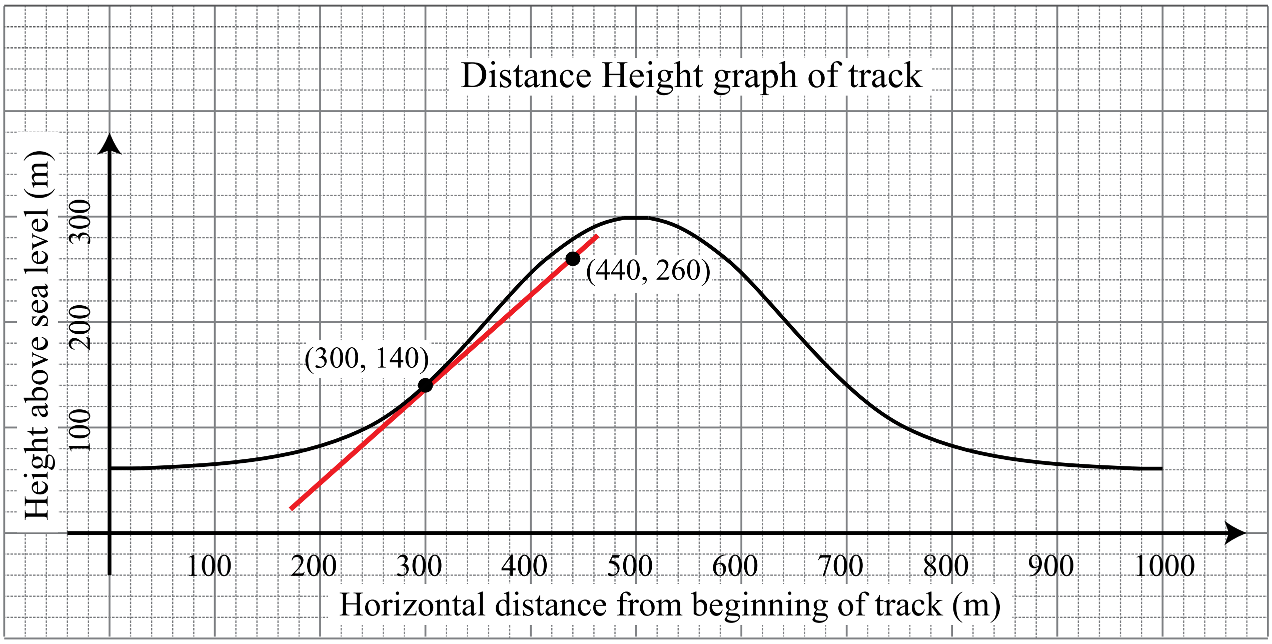
(b) Identify another point (x_2, y_2) on the tangent line

(c) Calculate the gradient of the tangent line using the formula:

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: The gradient of the track at 300 metres along is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{260 - 140}{440 - 300} = \frac{120}{140} = 0.857$$ which can be rounded to 0.9



5. Without calculating, can you guess what the gradient will be at 700 metres into the track? Explain your answer.

Note: This is one of many ways of calculating the gradient of a straight line

Answer: The gradient will be −0.9 because the graph is symmetrical so the slope will be the same, only negative as it is going downhill.

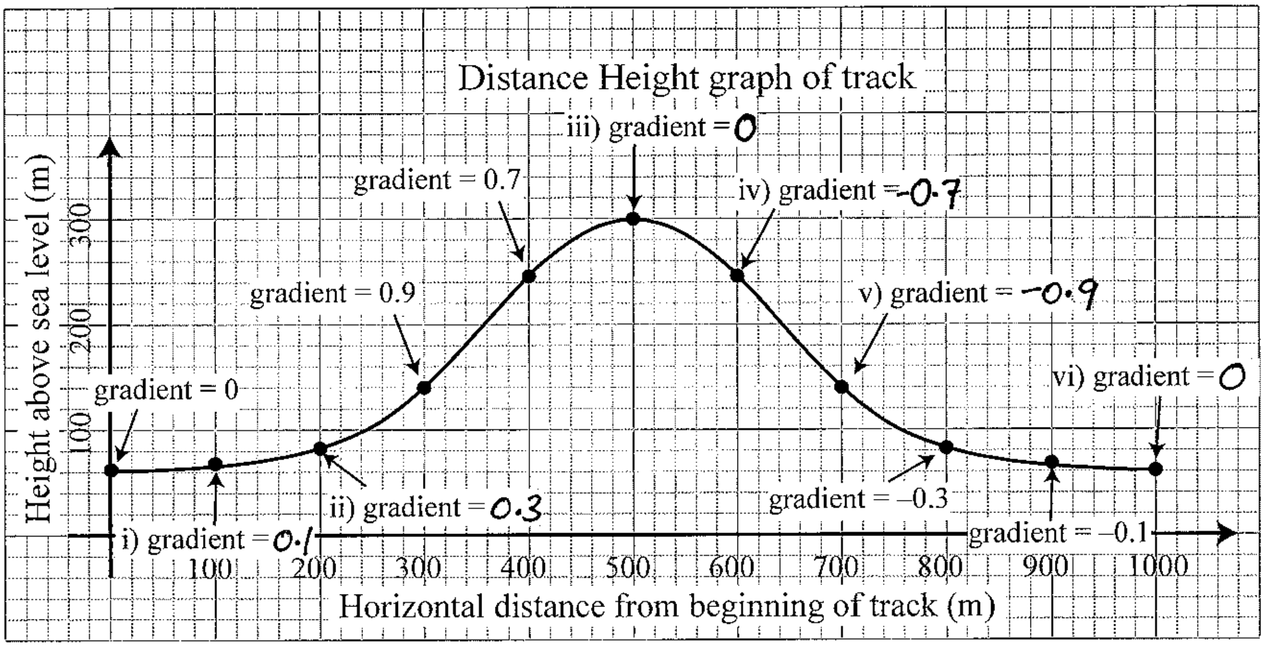
6. What does the track look like when the gradient of the track is zero?

Answer: Flat

7. What does the track look like when the gradient of the track is negative?

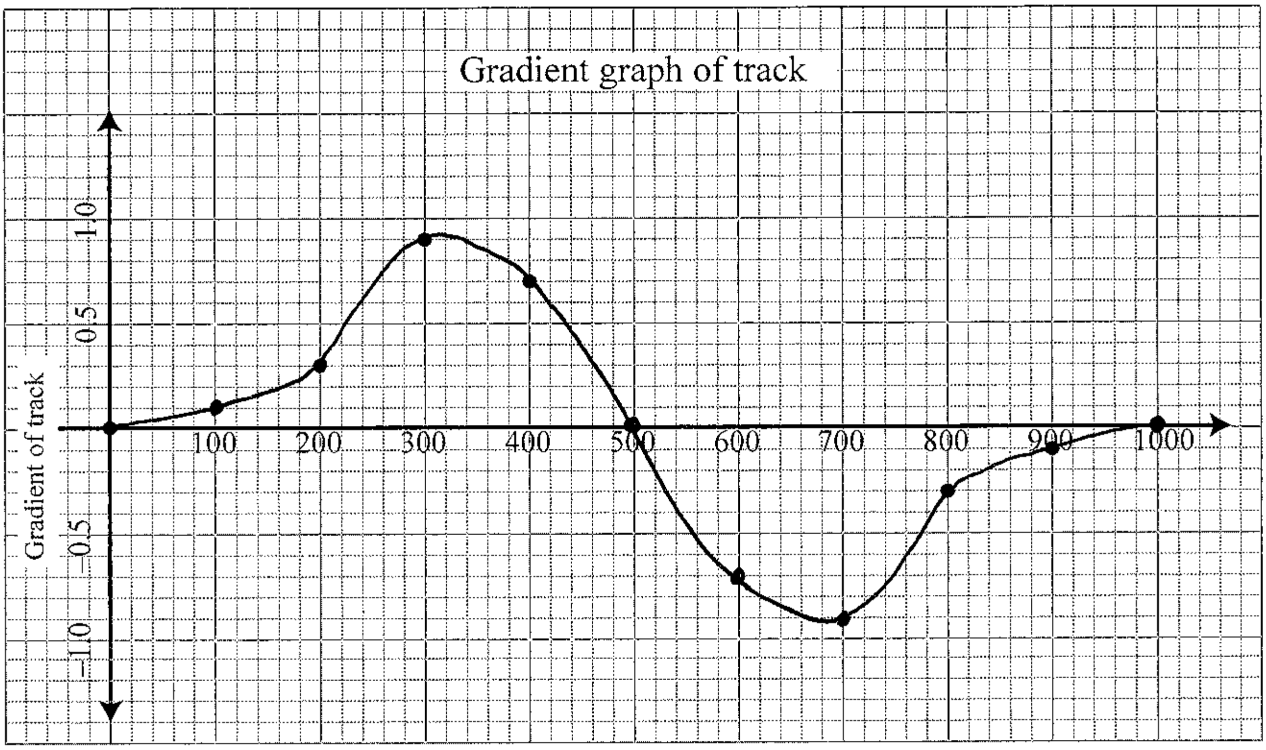
Answer: Downhill

8. The gradients have been worked out for some points on the graph. Fill in the remaining ones.



Answer: i) 0.1; ii) 0.3; iii) 0; iv) ⁻0.7; v) ⁻0.9; vi) 0

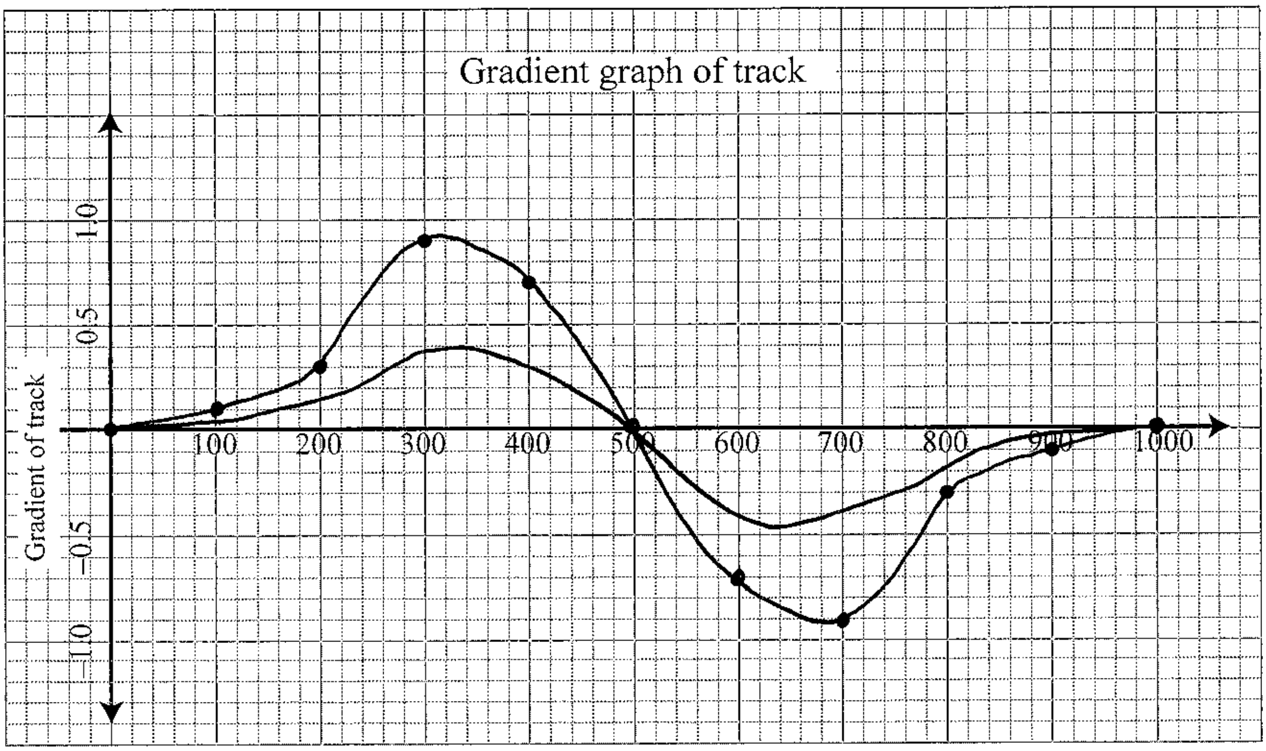
9. On the axes below, some gradients of the track have been plotted. Plot the missing gradients of the track and join them with a smooth curve.



10. The graph you have drawn is the *gradient graph* of the tramping track. What does it tell you about the tramping track?

Answer: The slope/gradient of the tramping track

11. Below is the *distance–height graph* of another tramping track. Without doing any calculations, what would the *gradient graph* look like for this tramping track? Draw your answer on the same axes as your answer to question 9.



Task 1: The tramping problem

Overview: This open-ended task forms the basis for the rest of the booklet, and should not be skipped. Students should work in teams of three for 30 minutes to create a team solution. If there is time available, teams can present their methods to the rest of the class. Facilitate mathematical questioning from the audience, and enjoy the diversity of ideas students bring to the problem.

To celebrate their 40th wedding anniversary, Helen and Brendan O'Neill are planning a tramp with their children and grandchildren. The local park provided a *gradient graph* for a nearby 5 kilometre tramp, but the O'Neills want to check it is suitable for their needs. Helen wants to know if there is a summit where they can have lunch and enjoy the view, and Brendan wants to know where the tramping gets difficult.

The O'Neills need your help!

Design a method that the O'Neills can use to sketch a *distance-height graph* of the actual track. You can assume that the track begins at sea level.

Write a letter to the O'Neills explaining your method, and use your method to describe the tramping track whose gradient graph is given on the next page. In particular, you must show any peaks and valleys in the track, uphill and downhill portions of the track, and the steepest and easiest parts of the track.

Most importantly, your method needs to work not only for this tramping track, but also for any other tramping track the O'Neills might consider in the future. In your letter, demonstrate to the O'Neills how your method can be used for any other tramping track they might take.

Note: As the problem statement is quite long, it works well to have one or two students read it out loud, as students are more likely to pay attention when they know they might be called upon to read part of the problem statement.

Before students start working, ask the class three questions to check they know what they are being asked to do:

(1) Question: Who are your clients?
Answer: Helen and Brendan

(2) Question: What is their problem?
Answer: They want to know what the tramping track will look like.

(3) Question: What do they want you to give them?"
Answer: A ***letter*** that describes a ***method*** for figuring out the ***distance-height graph*** of any tramping track from its gradient graph.

Here are some frequently asked questions about implementing these open-ended tasks

1. Should I provide resources (e.g. calculators, counters, etc.) for the students? How should I provide them?

- After launching the problem statement, say to the class: “You can use anything you have with you, anything you can find in the class, or you can ask me for equipment and I will see what I can find”
- Try not to prioritise one piece of equipment at the beginning, as students might think they have to use it to find the “correct” answer.
- You can put resources on group tables at the beginning of the task, but emphasise that these are just a selection of tools they might find useful, they don’t have to use them, and they can use other tools if they wish.
- Alternatively, you could have a collection of resources on a common table at the front of the class, with the same caveat.

2. What if a group of students don’t talk to each other?

- Use 1-minute of thinking time for the whole class before starting the task. After launching the problem statement, say to the class: “Take 1 minute to think about this problem in silence”. This will give students an opportunity to come up with ideas before they are forced to share them.
- When interacting with small groups that aren’t talking, ask them to share what they are thinking with the other students in the group, not just with you. E.g., say “Sam, please tell Julia and Tina what ideas you’ve had about this problem”, not “Sam, tell me what you’re thinking”. Then encourage the other team mates to respond to each others’ ideas: “Julia, what do you think of Sam’s suggestion?”
- Observe nonverbal clues and make them explicit to start discussion. E.g., “Benny, you’re frowning. What does that mean? Tell the others in your team what you’re thinking”.
- Make it clear that it’s ok to disagree with each other. Tell them they should challenge each other, because that’s how their ideas will get better.

3. What if a group of students finish before the rest of the class?

- Ask if they have written their letter.
- If they have, ask if their letter is clear enough for the client to follow without needing to ask for help.
- If their letter is clear, and their solution is sound, invite them to present their solution to the rest of the class once everyone’s finished. Let them spend the remaining time preparing their presentation while the rest of the class finish writing their letters.

4. What if some students are stuck or on the wrong track, mathematically?

- Remember that it’s OK for students to experience being stuck. Sometimes it’s a necessary phase before a breakthrough.
- Try not to give them explicit solutions or hints. Instead, get them to test their method on some examples that will reveal the method’s flaws.
- Instead of telling them the answer, you could suggest they use a resource that will help see the problem in a new way. For example, you could say, “have you thought about using a ruler to draw vertical lines down?”

5. What if one student wants to do it all?

- In some cases, one or two students may take charge to the extent that they take over the problem. As a consequence, other team members may lose interest as they feel their contributions are not important. When this happens, remind the team that they all need to agree on the final method.Ask other team members whether they agree with the student's method, and whether they have other suggestions.
- Ask the team member(s) who have not contributed much to write up the final method.
- Tell the team they all need to be comfortable with explaining their method to the rest of the class.

6. What if some students are off task?

- Sometimes, taking a break can be a useful part of the modelling process. After a short break, students often return to the problem with fresh eyes, which can lead to a new way of interpreting the data.
- Are they off task because of communication issues, or because they have already finished, or because they are stuck?
- Remind the students they are responsible for producing a group letter.
- Remember that it's OK if some groups don't finish the task as well as you hoped. By struggling on the problem, they will be better prepared to appreciate good solutions that other groups present.

7. What if some students won't write a letter?

Getting students to write can always be a challenge. Some ways to scaffold this process are:

- Encourage them to write step-by-step instructions
- Encourage them to draw diagrams.
- Provide a writing frame like the one shown below:

Dear Helen and Brendan,

To figure out the distance-height graph of a tramping track, follow these steps:

Step 1:

Step 2:

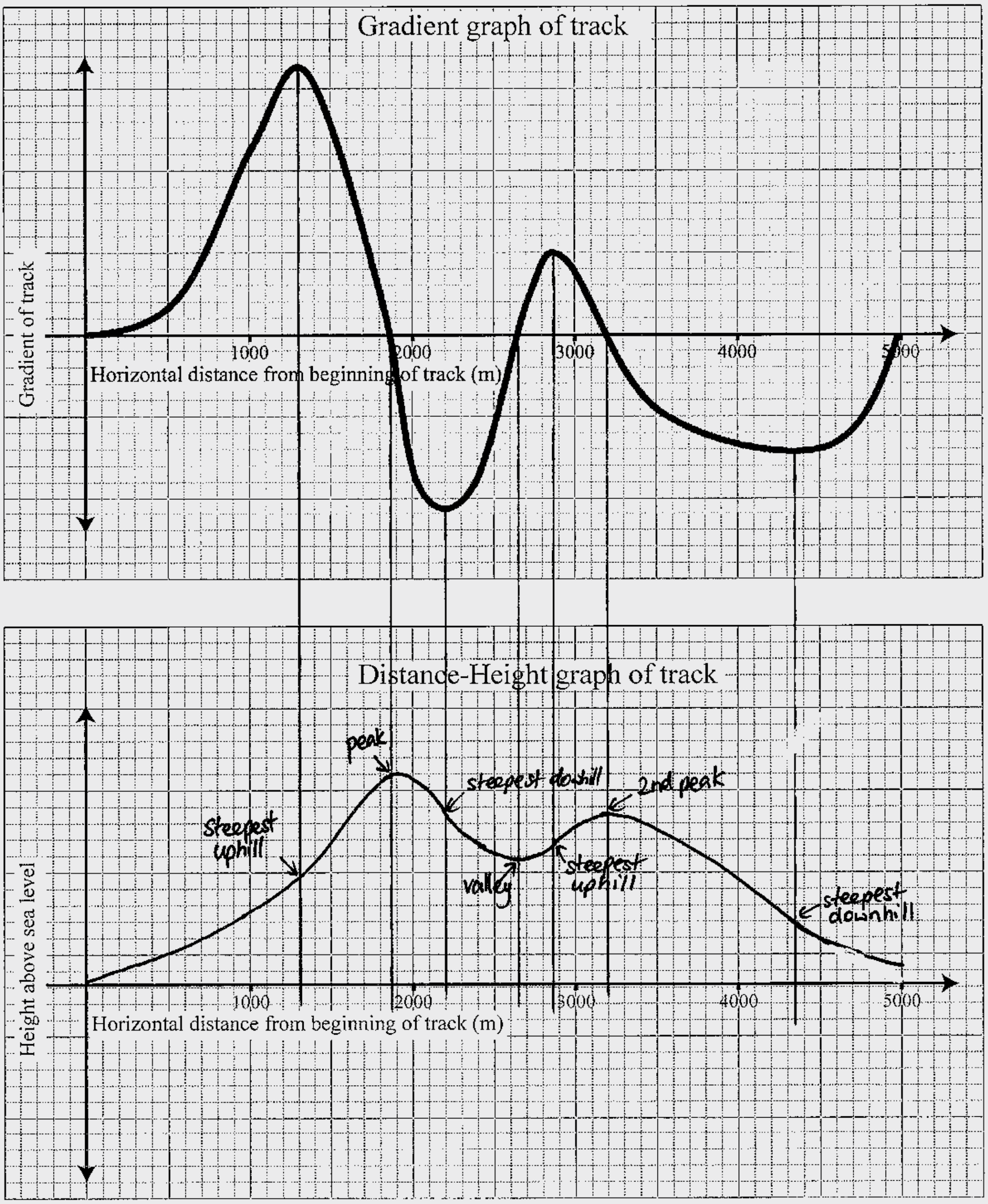
Draw a diagram to explain your method

Demonstrate your method with a different gradient graph.

Yours sincerely,

8. How do I mark the written communication of the letter?

Students may not have had much practice explaining mathematical methods in everyday English in the form of a letter. Some of the follow up tasks will be useful for enhancing students' written mathematical communication (e.g. Tasks 8 and 9), and the communication assessment task (Task 10) is an opportunity for students to demonstrate how their written mathematical communication has developed. Guidelines on how to assess the written mathematical communication of Task 10 are given at the end of this booklet, and can certainly be applied to the Tramping problem (Task 1) if you wish. However, it may be more productive to focus on strengthening students' written mathematical communication first through the follow up tasks before assessing their written communication, which is why we have not included the assessment guide in this task.



Sample answer:

Dear O'Neills,

- Uphill portions of the tramping track are indicated by positive values on the gradient graph.
- Downhill portions of the tramping track are indicated by negative values on the gradient graph.
- Peaks and valleys on the tramping track are indicated by x-axis intercepts on the gradient graph.
    - If the gradient graph crosses the x-axis from positive to negative values, then the x-axis intercept corresponds to a peak.
    - If the gradient graph crosses the x-axis from negative to positive, then the x-axis intercept corresponds to a valley.

Sincerely,