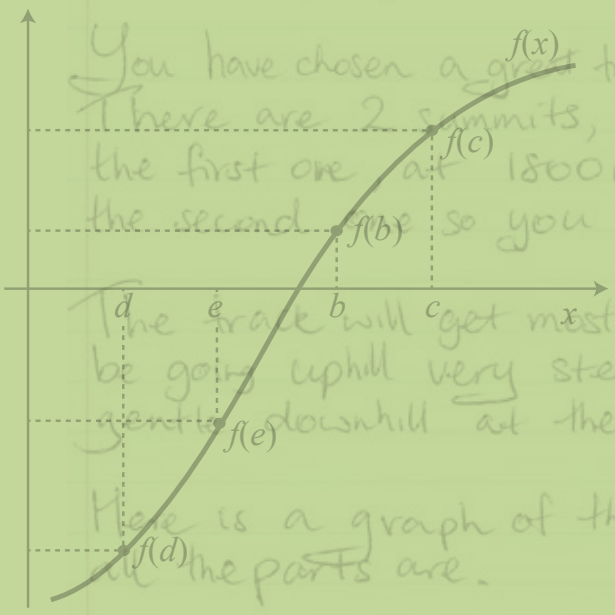
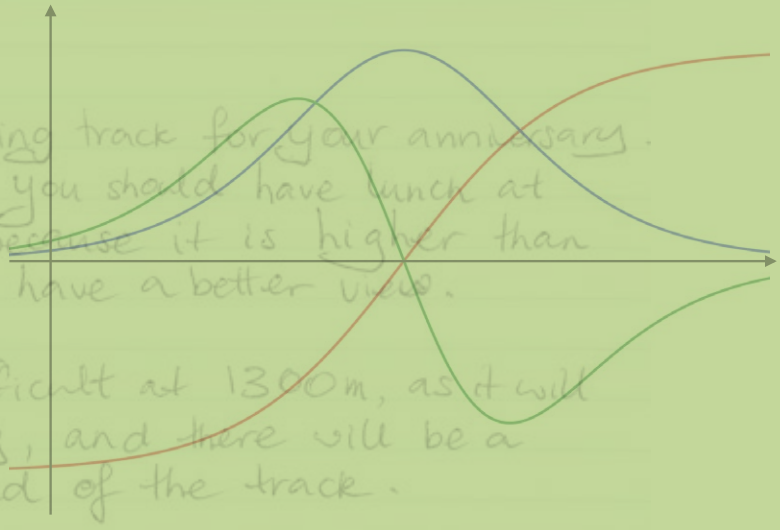
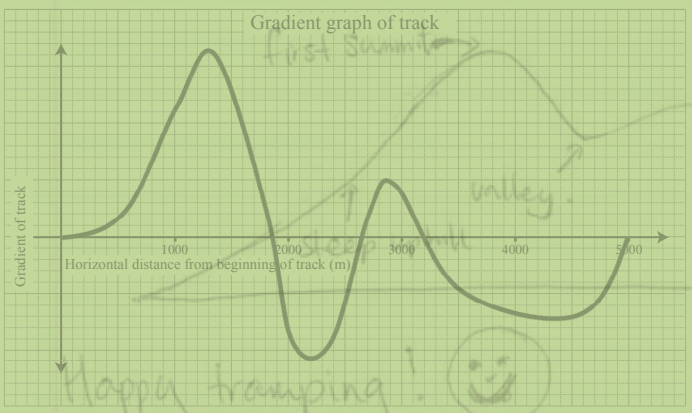
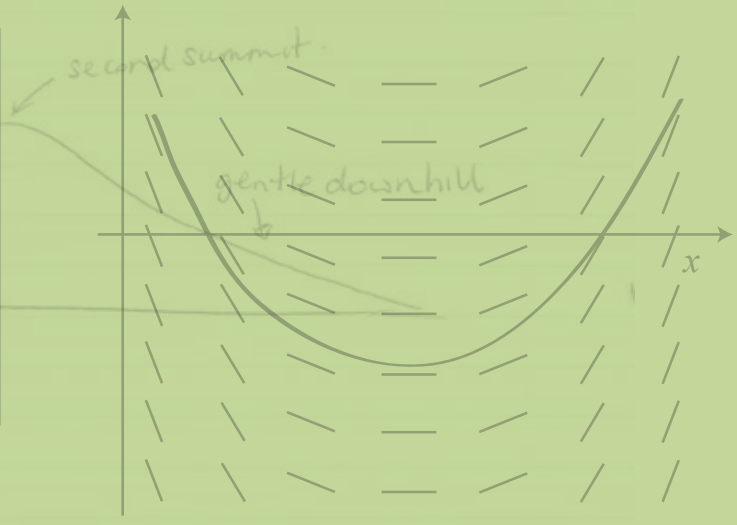
Graphical Antiderivatives

STUDENT BOOKLET









By Caroline Yoon, Tommy Dreyfus, Tessa Miskell and Mike Thomas

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Warm-up task

Watch the first minute of the slip n slide video and answer the following questions.

http://www.youtube.com/watch?v=DPp2HIIMkmU

Discuss with your classmates, the following questions:

- Use your left hand to show the slope of the slide.
- Why does the slide slope upwards at the end?
- Is there any part of the slide that is flat? If so, where?

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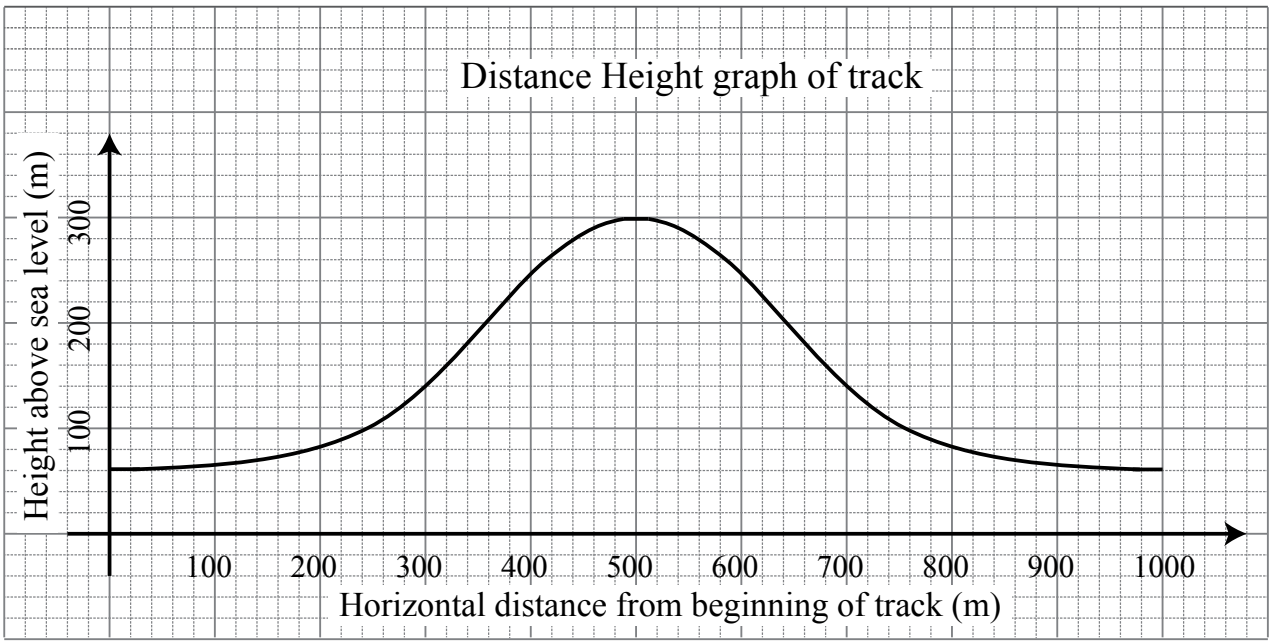
Below is a graph of a tramping track that goes up and down a hill.



1. What is the horizontal length of this track? (Note: this is different from the actual length along the track)

2. At what horizontal distance is the track the highest?

3. How high is the highest part of the track?

4. Where does the track appear to be steepest uphill?

We can measure how steep the track is at any point by finding the *gradient of the tangent line* to the graph of the track.

HOW TO: Find the gradient of the tangent line at a point

a) Draw a tangent line to the graph at a point (x_1, y_1)

b) Identify another point (x_2, y_2) on the tangent line

c) Calculate the gradient of the tangent line using the formula:

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

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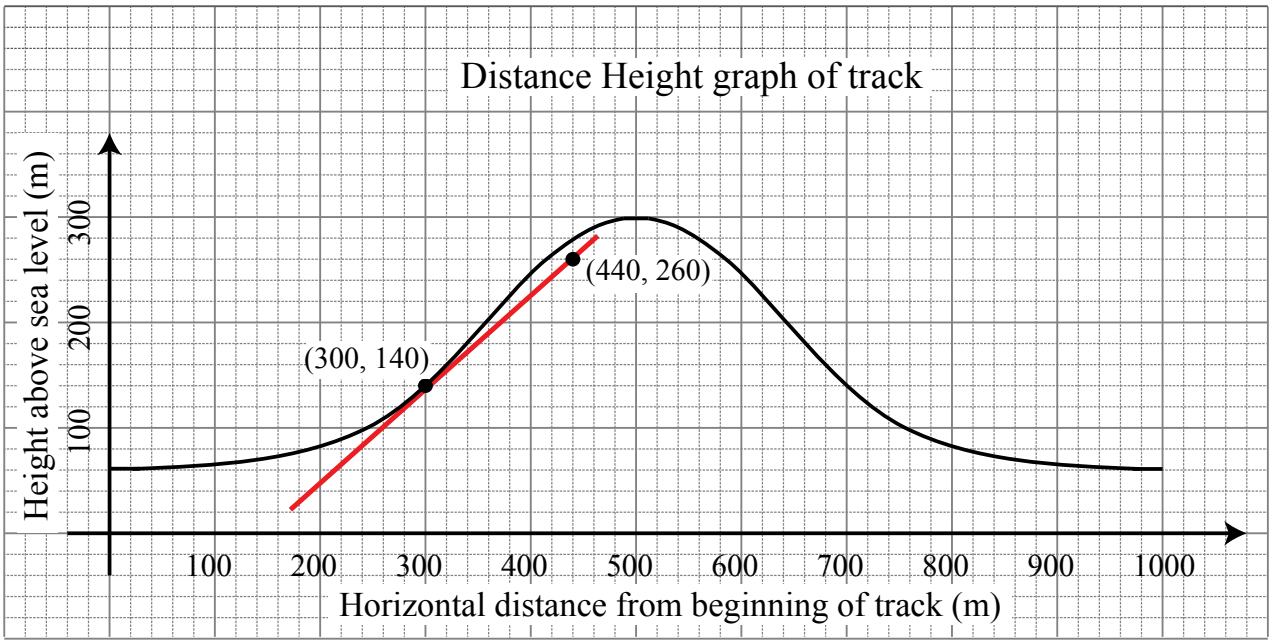
Example: The gradient of the track at 300 metres along is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{260 - 140}{440 - 300} = \frac{120}{140} = 0.857$ which can be rounded to 0.9

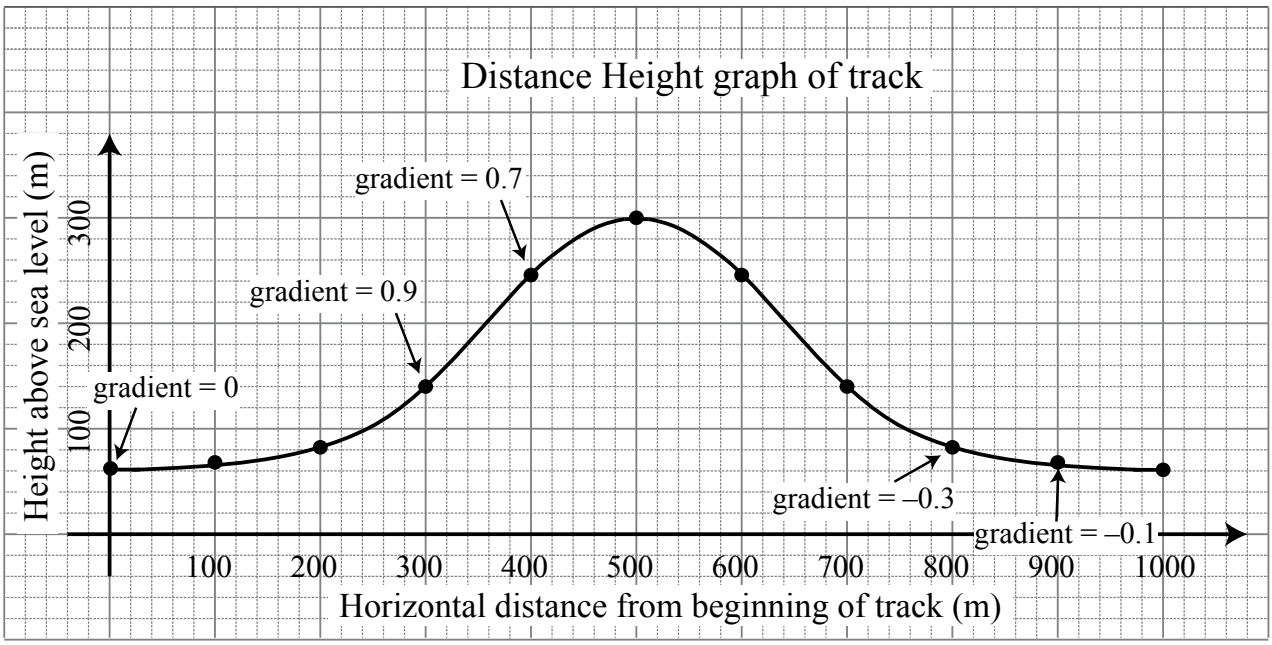


5. Without calculating, can you guess what the gradient will be at 700 metres into the track? Explain your answer.

6. What does the track look like when the gradient of the track is zero?

7. What does the track look like when the gradient of the track is negative?

8. The gradients have been worked out for some points on the graph. Fill in the remaining ones.



9. In your answer booklet, some gradients of the track have been plotted. Plot the missing gradients of the track and join them with a smooth curve.

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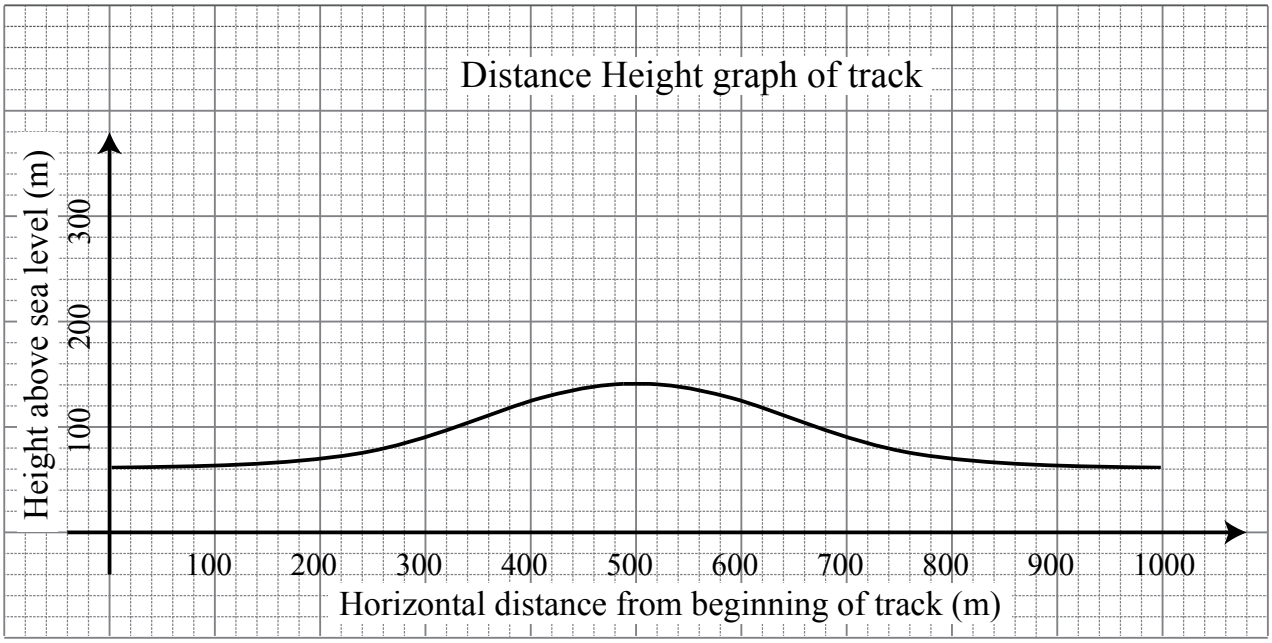
10. The graph you have drawn is the *gradient graph* of the tramping track. What does it tell you about the tramping track?

11. Below is the *distance-height graph* of another tramping track. Without doing any calculations, what would the *gradient graph* look like for this tramping track? Draw your answer on the same axes as your answer to question 9.



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Task 1: The tramping problem

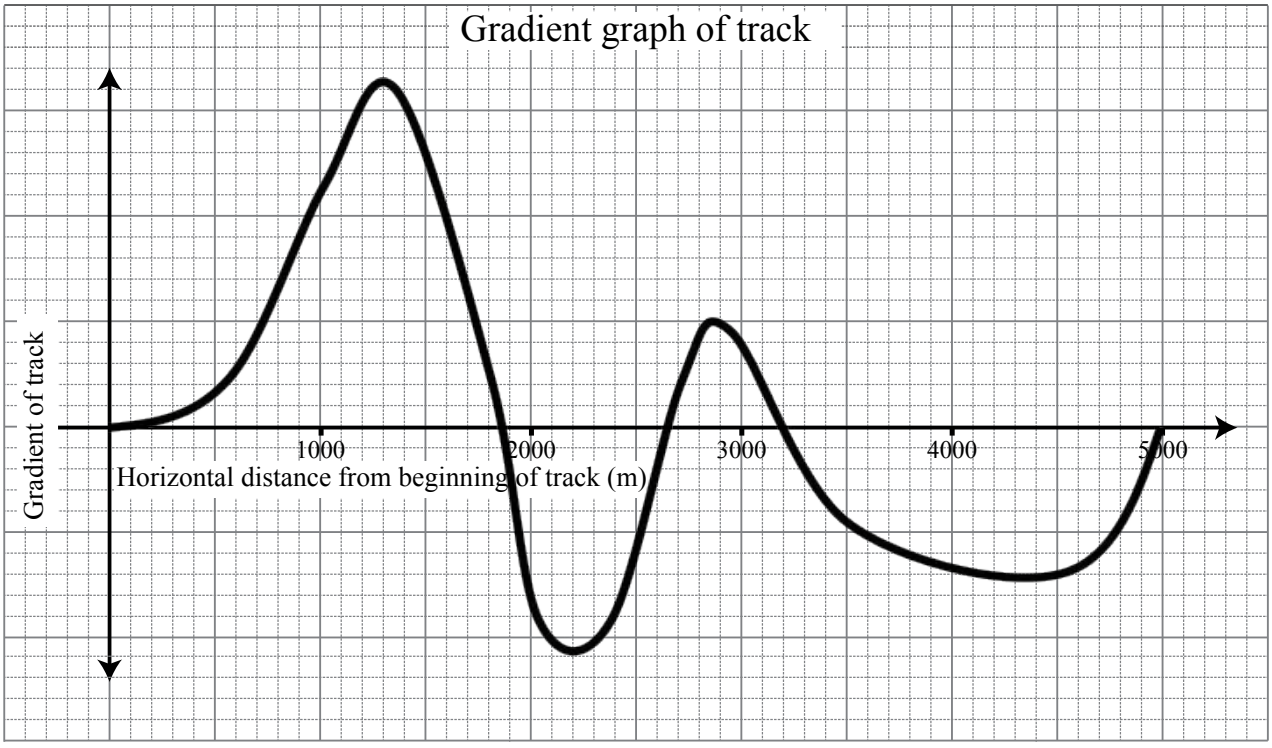
To celebrate their 40th wedding anniversary, Helen and Brendan O'Neill are planning a tramp with their children and grandchildren. The local park provided a *gradient graph* for a nearby 5 kilometre tramp, but the O'Neills want to check it is suitable for their needs. Helen wants to know if there is a summit where they can have lunch and enjoy the view, and Brendan wants to know where the tramping gets difficult.

The O'Neills need your help!

Design a method that the O'Neills can use to sketch a *distance-height graph* of the actual track. You can assume that the track begins at sea level.

Write a letter to the O'Neills explaining your method, and use your method to describe the tramping track whose gradient graph is given below. In particular, you must show any peaks and valleys in the track, uphill and downhill portions of the track, and the steepest and easiest parts of the track.

Most importantly, your method needs to work not only for this tramping track, but also for any other tramping track the O'Neills might consider in the future. Within your letter, demonstrate to the O'Neills how your method can be used for any other tramping track they might take.



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