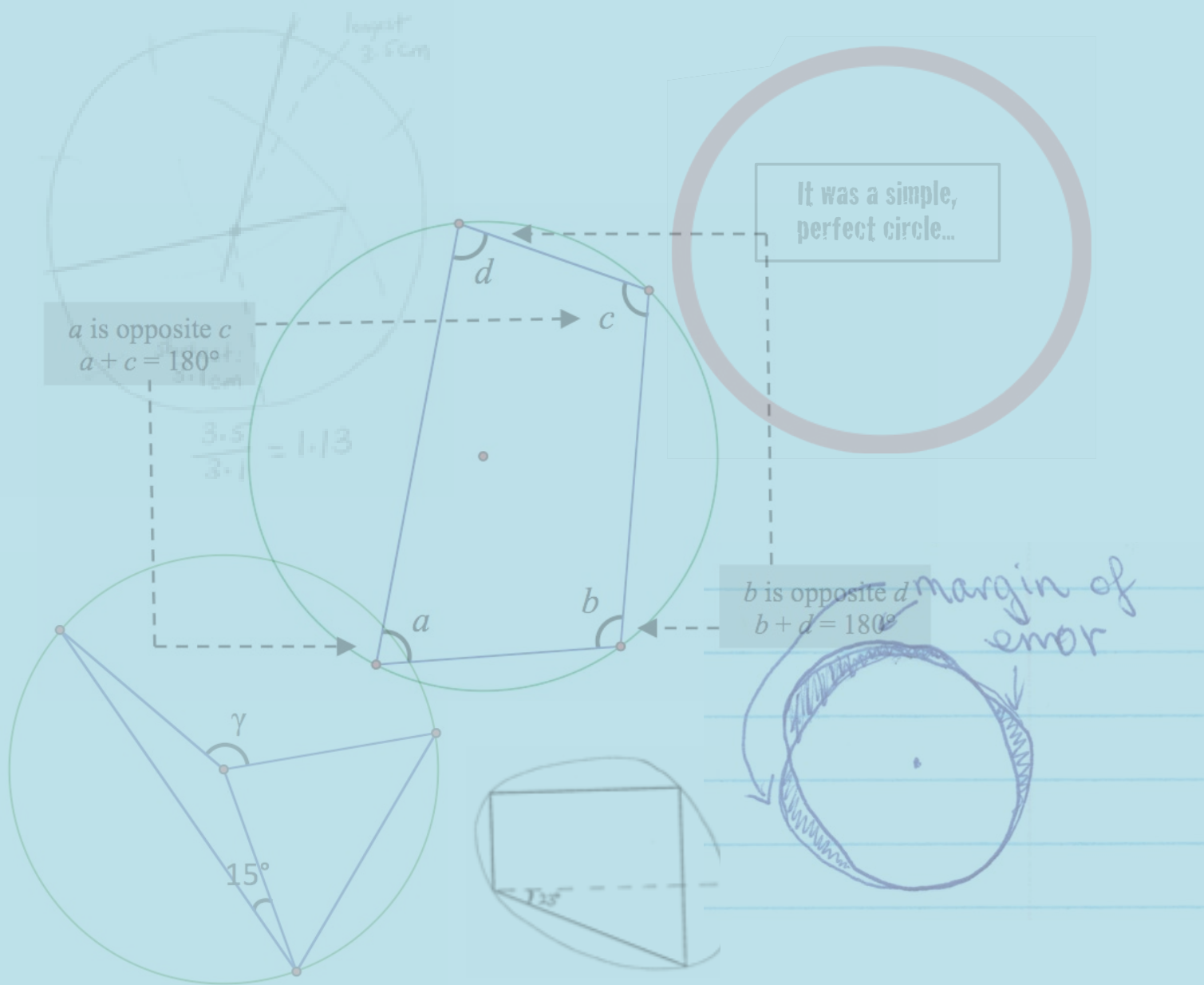


Circle Geometry

TEACHER MANUAL



By Peter Radonich and Caroline Yoon



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Introduction

Warm up

Overview: The Giotto comic tells the story of the artist, Giotto di Bondone, who is fabled for demonstrating his artistic genius by drawing a perfect, freehand circle. This comic sets the stage for the context of freehand circle drawing, which is the basis for Task 1.

This warmup can be done with the whole class either the day before Task 1, or at the beginning of the class session when the task is implemented. The benefit of doing the warmup the day before is that it will give students more time to work on their written letters for Task 1, and may leave some room for student presentations at the end. However, it also works well if the warmup is presented on the same day as the task, as long as the warmup is completed within 10 minutes to leave enough time for the Freehand Circle problem.

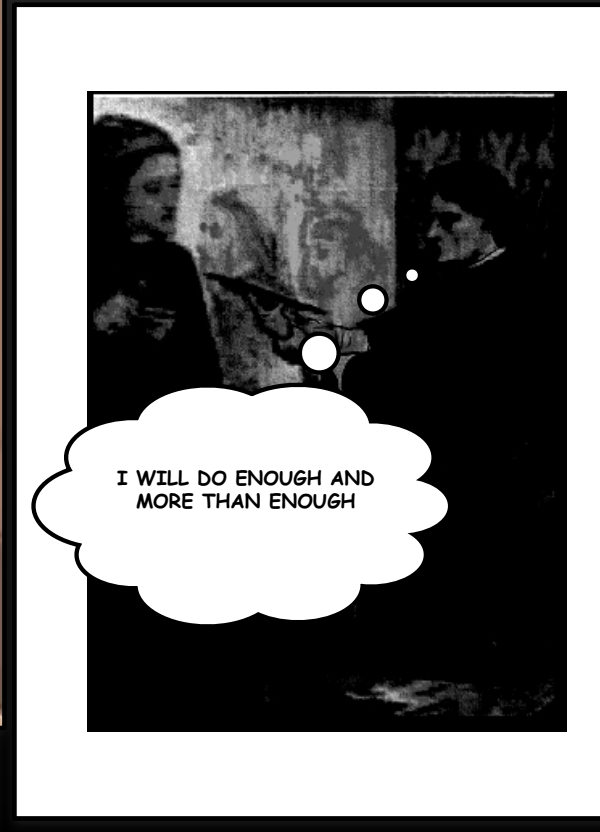


FOR THOSE WHO DON'T KNOW LATIN, THE POPE SAYS: "I NEED THE BEST ARTIST IN THE COUNTRY"

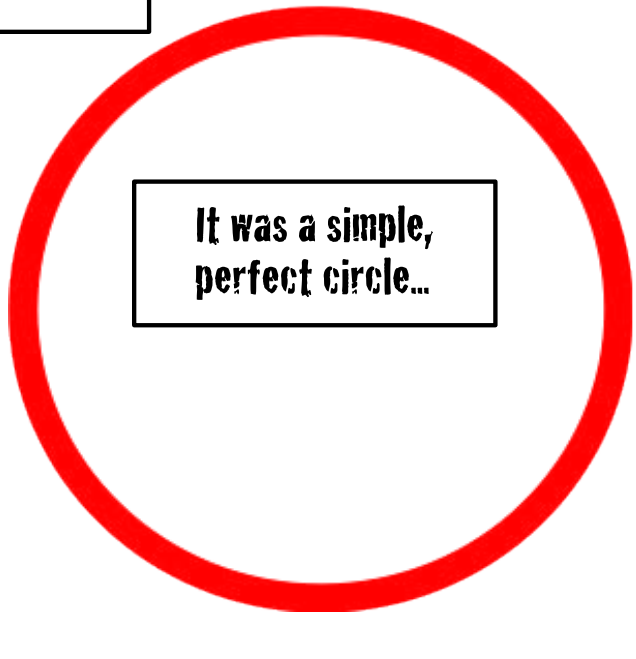
FOR THOSE WHO KNOW LATIN WELL... AN ONLINE TRANSLATOR WAS USED!



AFTER A LOT OF RIDING AND SEARCHING, THE MESSENGER FINDS GIOTTO... PAINTING A MAGNIFICENT PORTRAIT.



GIOTTO TOOK A BRUSH FROM A POT OF RED PAINT AND WITH A FLICK OF HIS WRIST, PUT HIS DESIGN ON THE MESSENGER'S PAPER



It was a simple, perfect circle...



IS THAT ALL YOU'RE GOING TO SEND HIS HOLINESS?!



THAT IS MORE THAN ENOUGH

TAKE IT WITH THE DRAWINGS THE OTHERS HAVE DONE AND SEE IF HIS HOLINESS UNDERSTANDS



LIKE A COMPASS...

IN AN INSTANT...

AFFER MIHI GIOTTO!



GIOTTO HAS TAKEN ME FOR A FOOL... WHAT WILL THE POPE MAKE OF THIS?

HE JUST FLICKED HIS ARM LIKE IT WAS A COMPASS... IT WAS DONE IN AN INSTANT.



THE POPE EXAMINED THE RED CIRCLE AND RECOGNISED IT WAS THE WORK OF GENIUS. HE KNEW IT WAS GIOTTO HE MUST COMMISSION TO DO THE ART WORK IN THE BASILICA.

1. Read the comic about Giotto and the Pope

(a) Why did the pope want to find an artist?

Note: These questions are designed to check that students are engaged in the context of the problem. They are best suited to a whole class discussion, rather than individual written responses.

Answer: He needed someone to paint his church

(b) What did Giotto give the messenger?

Answer: A perfect freehand circle

(c) Why did the pope choose Giotto as his artist?

Answer: He recognised that only a genius could draw a perfect freehand circle

2. In your group, make TWO attempts EACH at drawing a perfect circle. Discuss in your group, whose circle is the most perfect.

(a) Is it easier to draw big or small circles? Why?

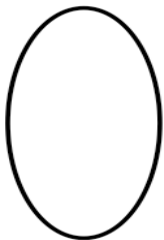
Note: Students relish the opportunity to try and draw their own freehand circles. If they don't draw them now, they will draw them during task 1, which could disrupt the flow of the activity.

Answer: Both have their advantages and disadvantages: Small circles require a smaller radius, so you have greater control, whereas big circles are more difficult to control. On the other hand, mistakes are more prominent in small circles than they are in large circles.

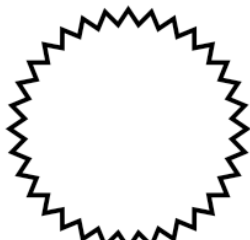
(b) What are some words that you associate with circles?

Answer: Circumference, centre, round, smooth, etc.

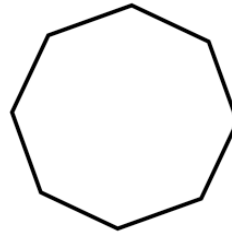
(c) Which of these figures is MOST like a circle? Which is LEAST?



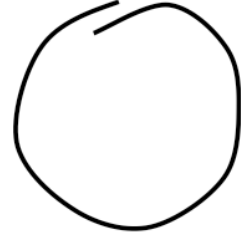
A



B



C



D

Answer: All of the figures have some circle features and some non-circle features:

A is smooth, but is not a circle – it is an ellipse

B looks circular from far away, but it is not smooth

C is a regular octagon and is not smooth

D looks circular from far away, but it is an open curve

3. Watch the following video: <http://www.youtube.com/watch?v=eAhfZUZiwSE>

World Freehand Circle Drawing Champion



Task 1: The Freehand Circle problem

Overview: This open-ended task forms the basis for the rest of the booklet, and should not be skipped. Students should work in teams of three for 30 minutes to create a team solution. If there is time available, teams can present their methods to the rest of the class. Facilitate mathematical questioning from the audience, and enjoy the diversity of ideas students bring to the problem.

Official Circle Drawing Competition

Highbury Pancake House will hold a Circle Drawing Competition!
Come along to win the title of
Highbury's Circle Drawing Champion of 2008!
The winner will receive free pancakes for a year.



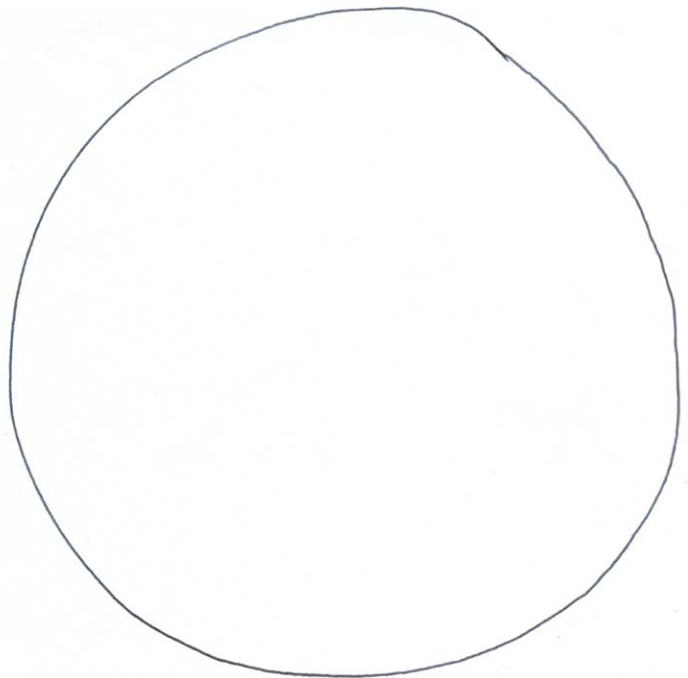
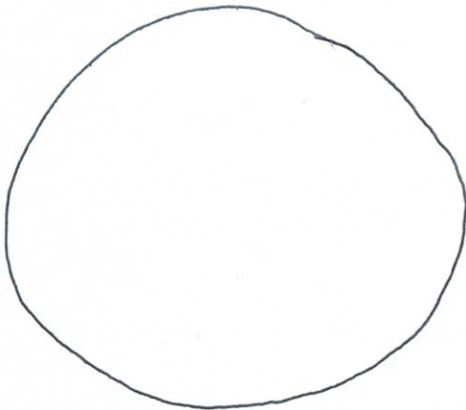
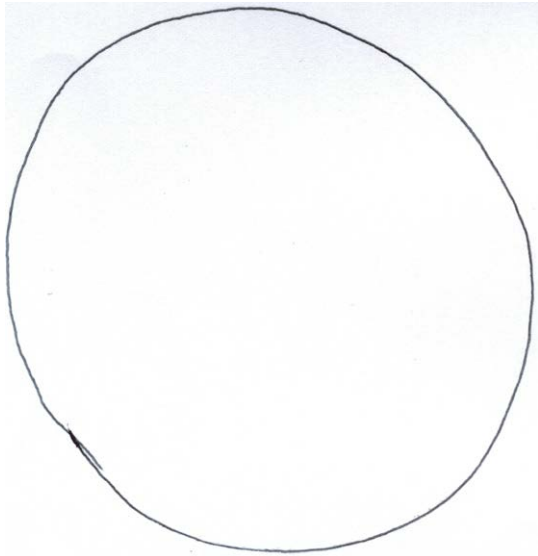
Rules, Terms, and Conditions:

Judge's decision is final; no correspondence will be entered into. Highbury Pancake House employees are ineligible to enter. Circles must be bigger than 15 centimetres wide. Circles must be drawn in under 5 seconds. Circles must be drawn on A3 paper. 1 attempt per person.

Bonnie, the organiser of the Circle Drawing Competition, needs your help. Last year, she was accused of being biased because her best friend won. This year, she has decided to make the judging process as fair as possible so no one complains.

Your task is to create a method for ranking circle attempts from the most circular to the least circular. Use your method to rank the circle attempts you have been given. In addition, your method must work for any other circle attempts that might be drawn on the night of the competition. **Write a letter to Bonnie, in which you explain your team's method and show how to use it to rank circle attempts from most to least circular.**

Test your method on these circle attempts:



Note: As the problem statement is quite long, it works well to have one or two students read it out loud, as students are more likely to pay attention when they know they might be called upon to read part of the problem statement.

Before students start working, ask the class three questions to check they know what they are being asked to do:

(1) Question: Who's your client?

Answer: Bonnie

(2) Question: What is Bonnie's problem?

Answer: She needs a fair way of ranking circle attempts

(3) Question: What does she want you to give her?"

Answer: A **letter** that describes a **method**, not just the answer, and the method **needs to work for any circle attempts**.

Here are some frequently asked questions about implementing these open-ended tasks

1. Should I provide resources (e.g. calculators, counters, etc.) for the students? How should I provide them?

- After launching the problem statement, say to the class: "You can use anything you have with you, anything you can find in the class, or you can ask me for equipment and I will see what I can find". Ask them not to use google.
- Try not to prioritise one piece of equipment at the beginning, as students might think they have to use it to find the "correct" answer.
- You can put resources on group tables at the beginning of the task, but emphasise that these are just a selection of tools they might find useful, they don't have to use them, and they can use other tools if they wish.
- Alternatively, you could have a collection of resources on a common table at the front of the class, with the same caveat.
- Some resources that are useful to have on hand are: rulers, string, scissors, compasses, protractors.

2. What if the members of a group don't talk to each other?

- Use 1-minute of thinking time for the whole class before starting the task. After launching the problem statement, say to the class: "Take 1 minute to think about this problem in silence". This will give students an opportunity to come up with ideas before they are forced to share them.
- When interacting with small groups that aren't talking, ask them to **share what they are thinking with the other students in the group, not just with you**. E.g., say "Sam, please tell Julia and Tina what ideas you've had about this problem", not "Sam, tell me what you're thinking". Then encourage the other team-mates to respond to each others' ideas: "Julia, what do you think of Sam's suggestion?"
- Observe nonverbal clues and make them explicit to start discussion. E.g., "Benny, you're frowning. What does that mean? Tell the others in your team what you're thinking".
- Make it clear that it's OK to disagree with each other. Tell them they should challenge each other, because that's how their ideas will get better.

3. What if a group of students finishes before the rest of the class?

- Ask if they have written their letter.
- If they have, ask if their letter is clear enough for the client to follow without needing to ask for help.
- If their letter is clear, and their solution is sound, invite them to present their solution to the rest of the class once everyone's finished. Let them spend the remaining time preparing their presentation while the rest of the class finish writing their letters.

4. What if some students are stuck or on the wrong track, mathematically?

- Remember that it's OK for students to experience being stuck. Sometimes it's a necessary phase before a breakthrough.
- Try not to give them explicit solutions or hints. Instead, get them to test their method on some examples that will reveal the method's flaws. For example, you could draw a square and ask – “would your method work on that?”
- Instead of telling them the answer, you could suggest they use a resource that will help see the problem in a new way. For example, you could say, “have you thought about using a compass?”

5. What if one student wants to do it all?

- In some cases, one or two students may take charge to the extent that they take over the problem. As a consequence, other team members may lose interest as they feel their contributions are not important. When this happens, remind the team that they all need to agree on the final method.
- Ask other team members whether they agree with the student's method, and whether they have other suggestions.
- Ask the team member(s) who have not contributed much to write up the final method.
- Tell the team they all need to be comfortable with explaining their method to the rest of the class.

6. What if some students are off task?

- Sometimes, taking a break can be a useful part of the modelling process. After a short break, students often return to the problem with fresh eyes, which can lead to a new way of interpreting the data.
- Are they off task because of communication issues, or because they have already finished, or because they are stuck?
- Remind the students they are responsible for producing a group letter.
- Remember that it's OK if some groups don't finish the task as well as you hoped. By struggling on the problem, they will be better prepared to appreciate good solutions that other groups present.

7. What if some students won't write a letter?

- Getting students to write can always be a challenge. Some ways to scaffold this process are:
 - Encourage them to write step-by-step instructions
 - Encourage them to draw diagrams.
 - Provide a writing frame like the one shown below:

Dear Bonnie,

Here is a method for ranking freehand circles.

Step 1:

Step 2:

Draw a diagram to explain your method

Demonstrate your method with some freehand circles

Yours sincerely,

8. How do I mark the written communication of the letter?

Students may not have had much practice explaining mathematical methods in everyday English in the form of a letter. Some of the follow up tasks will be useful for enhancing students' written mathematical communication (e.g. Tasks 8 and 9), and the final assessment task (Task 10) is an opportunity for students to demonstrate how their written mathematical communication has developed. Guidelines on how to assess the written mathematical communication of Task 10 are given at the end of this booklet, and can certainly be applied to the Freehand Circle problem (Task 1) if you wish. However, it may be more productive to focus on strengthening students' written mathematical communication first before giving them an assessment, which is why we have not included the assessment guide in this task.

Comment on answers: We have seen dozens of different, plausible solutions to this problem. One appropriate method finds the largest distance between two points on the perimeter of any circle attempt, then calculates the circumference of a perfect circle with this length as its diameter. A comparison between the ideal circumference and the actual circumference (measured by string and ruler) yields a measure of the circle attempt's circularity. But there are also many other appropriate methods.