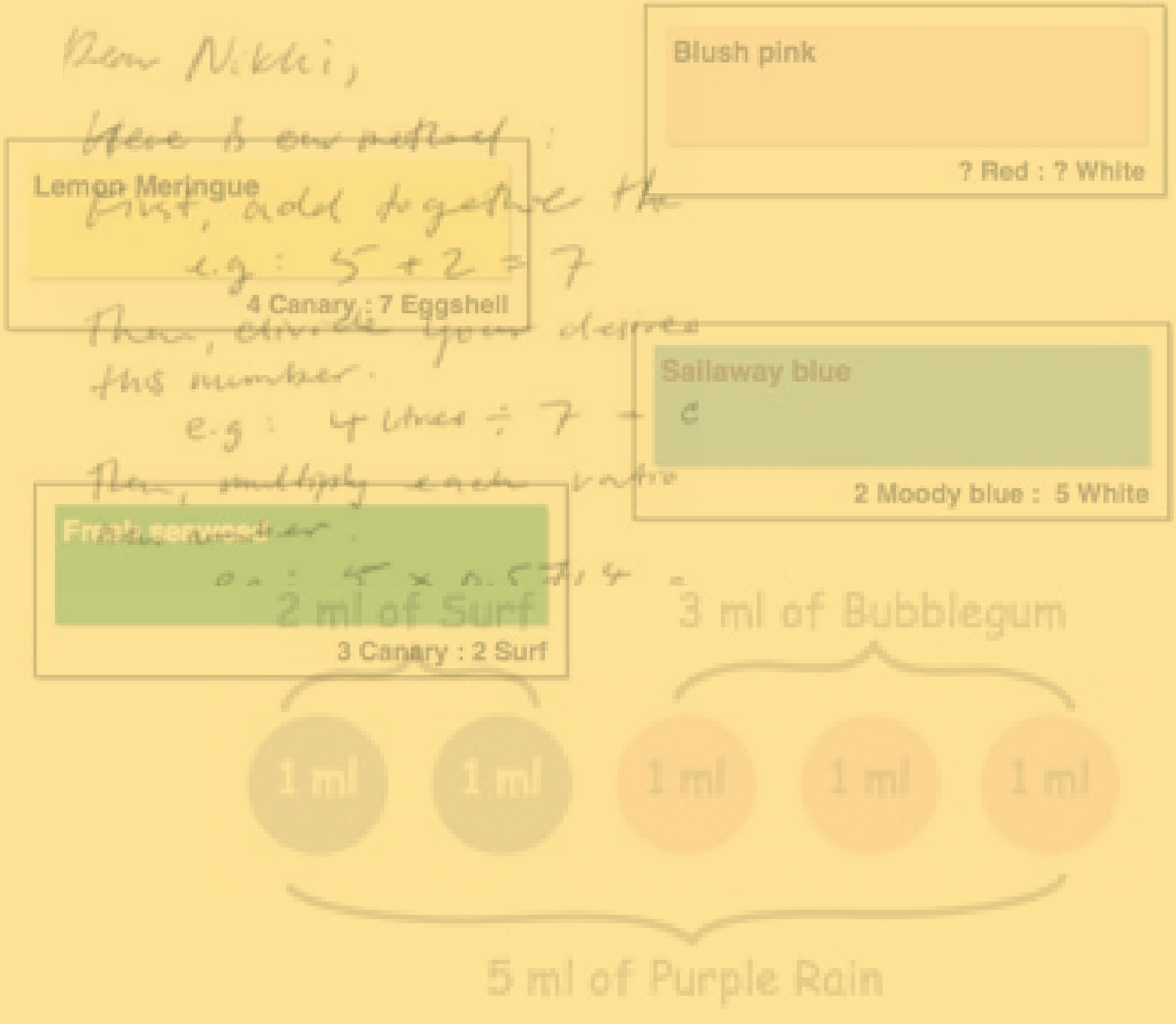
Equivalent Proportions

TEACHER MANUAL



By Caroline Yoon, Nikki Sullivan and Anne Patel

The LEMMA series:
Learning Encounters with
Meta-Mathematical Activities

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United Nations
Educational, Scientific and
Cultural Organization

New Zealand National
Commission for UNESCO
Te Kōmihana Matua o Aotearoa mō UNESCO

Contents

Introduction

Warm-up task

Overview: The goal of this warm-up is to prepare students for Task 1 (The Rothko problem), by engaging them in the context and mathematical concepts. It takes 10 minutes, and can be done with the whole class either the day before Task 1, or at the beginning of the class session when Task 1 is implemented. The benefit of doing the warm-up the day before is that it will give students more time to work on their written letters for Task 1, and may leave some room for student presentations at the end. However, it also works well if the warm-up is presented on the same day as Task 1, as long as the warm-up is completed within 10 minutes to leave enough time for the task.

What is this painting worth?

View the Rothko powerpoint to find out how much someone paid for it.

Note: The questions in the Rothko powerpoint are designed to check that students have engaged in the context of the problem. Download the powerpoint from www.nzcer.org.nz/LEMMA

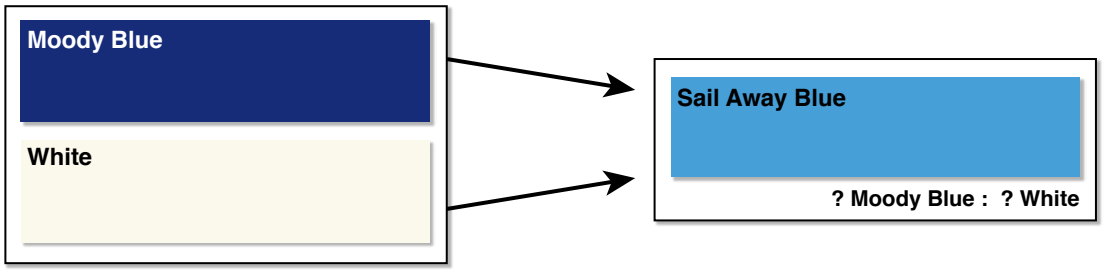
15-year-old Nikki Lim loves Rothko's paintings, but can't afford to buy one.

Being an enterprising and artistic teenager, she decides to create one in a similar style by painting large blocks of colour directly onto her bedroom wall.

Nikki finds a can of Moody Blue paint and a can of White paint in her garage.

She decides to mix Moody Blue and White to make a light shade of blue called Sail Away Blue, which she saw in a catalogue but cannot afford to buy.

1. How much *Moody Blue* and *White* paint do you think go into *Sail Away Blue*? Discuss with your group and write down your group's prediction as a ratio *Moody Blue* paint *: White* paint



Note: Give students an opportunity to guess the proportions of Moody Blue and White paint in Sail Away Blue. The actual ratio is 5 White : 2 Moody Blue. Even though very few students will guess the correct ratio, the act of guessing encourages them to think about ratios of paint colours, which is vital for understanding Task 1. Many students guess using percentages or fractions rather than ratios. For example, they may say "60% Blue, 40% White" or "2/3 Blue, 1/3 White". Classroom discussion and questions (a), (b) and (c) can be used to refresh students' knowledge of ratios.

(a) Albert guessed **half** *Moody Blue* and **half** *White*. Write this as a ratio.

Answer: 1 Moody Blue : 1 White

(b) Hilary guessed $\frac{2}{3}$ *Moody Blue* and $\frac{1}{3}$ *White*. Write this as a ratio.

Answer: 2 Moody Blue : 1 White

(c) Liam guessed **60%** *Moody Blue* and **40%** *White*. Write this as a ratio.

Answer: 3 Moody Blue : 2 White

Task 1: The Rothko problem

Overview: This open-ended task forms the basis for the rest of the booklet, and should not be skipped. Students should work in teams of three for 30 minutes to create a team solution. If there is time available, teams can present their methods to the rest of the class. Facilitate mathematical questioning from the audience, and enjoy the diversity of ideas students bring to the problem.

Nikki needs to make 4 litres of *Sail Away Blue* paint for her Rothko mural.

Nikki needs your help!

Create a method for figuring out how much *Moody Blue* paint and *White* paint Nikki needs to mix together to make 4 litres of *Sail Away Blue*.

Write a letter to Nikki, describing your method. Explain how she can use your method to make **any amount** of *Sail Away Blue* in the future. Also, describe how Nikki can adapt your method to make any amount of any other colour with different proportions, in case she changes her mind.

Note: As the problem statement is quite long, it works well to have one or two students read it out loud, as students are more likely to pay attention when they know they might be called upon to read part of the problem statement.

Before students start working, ask the class three questions to check they know what they are being asked to do:

(1) Question: Who's your client?
Answer: Nikki Lim

(2) Question: What is Nikki's problem?
Answer: She wants to make 4 litres of Sail Away Blue

(3) Question: What does she want you to give her?"
Answer: A ***letter*** that describes a ***method***, not just the answer, and the ***method needs to work for any ratios of paint.***

Here are some frequently asked questions about implementing these open-ended tasks

1. Should I provide resources (e.g. calculators, counters, etc.) for the students? How should I provide them?

- After launching the problem statement, say to the class: "You can use anything you have with you, anything you can find in the class, or you can ask me for equipment and I will see what I can find."
- Try not to prioritise one piece of equipment at the beginning, as students might think they have to use it to find the "correct" answer.
- You can put resources on group tables at the beginning of the task, but emphasise that these are just a selection of tools they might find useful, they don't have to use them, and they can use other tools if they wish.
- Alternatively, you could have a collection of resources on a common table at the front of the class, with the same caveat.

2. What if students in a group don't talk to each other?

- Use 1 minute of thinking time for the whole class before starting the task. After launching the problem statement, say to the class: "Take 1 minute to think about this problem in silence". This will give students an opportunity to come up with ideas before they are forced to share them.
- When interacting with small groups that aren't talking, ask them to share what they are thinking with the other students in the group, not just with you. E.g., say "Sam, please tell Julia and Tina what ideas you've had about this problem", not "Sam, tell me what you're thinking". Then encourage the other team mates to respond to each others' ideas: "Julia, what do you think of Sam's suggestion?"
- Observe nonverbal clues and make them explicit to start discussion. E.g., "Benny, you're frowning. What does that mean? Tell the others in your team what you're thinking".
- Make it clear that it's OK to disagree with each other. Tell them they should challenge each other, because that's how their ideas will get better.

3. What if a group of students finishes before the rest of the class?

- Ask if they have written their letter.
- If they have, ask if their letter is clear enough for the client to follow without needing to ask for help.
- If their letter is clear, and their solution is sound, invite them to present their solution to the rest of the class once everyone's finished. Let them spend the remaining time preparing their presentation while the rest of the class finishes writing their letters.

4. What if some students are stuck or on the wrong track, mathematically?

- Remember that it's OK for students to experience being stuck. Sometimes it's a necessary phase before a breakthrough.
- Try not to give them explicit solutions or hints. Instead, get them to test their method on some examples that will reveal the method's flaws. For example, you could draw a square and ask, "Would your method work on that?"
- Instead of telling them the answer, you could suggest they use a resource that will help see the problem in a new way. For example, you could say, "Have you thought about using counters?"

5. What if one student wants to do it all?

- In some cases, one or two students may take charge to the extent that they take over the problem. As a consequence, other team members may lose interest as they feel their contributions are not important. When this happens, remind the team that they all need to agree on the final method.
- Ask other team members whether they agree with the student's method, and whether they have other suggestions.
- Ask the team member(s) who have not contributed much to write up the final method.
- Tell the team they all need to be comfortable with explaining their method to the rest of the class.

6. What if some students are off task?

- Sometimes, taking a break can be a useful part of the modelling process. After a short break, students often return to the problem with fresh eyes, which can lead to a new way of interpreting the data.
- Are they off task because of communication issues, or because they have already finished, or because they are stuck?
- Remind the students they are responsible for producing a group letter.
- Remember that it's OK if some groups don't finish the task as well as you hoped. By struggling on the problem, they will be better prepared to appreciate good solutions that other groups present.

7. What if some students won't write a letter?

Getting students to write can always be a challenge. Some ways to scaffold this process are:

- Encourage them to write step-by-step instructions.
- Encourage them to draw diagrams.
- Provide a writing frame like the one shown below:

Dear Nikki,

This is what you need to do to figure out the amount of Moody Blue paint and White paint in 4 litres of Sail Away Blue.

Step 1:

Step 2:

Draw a diagram to explain your method

Demonstrate your method on a different combination of paints.

Yours sincerely,

8. How do I mark the written communication of the letter?

- Students may not have had much practice explaining mathematical methods in everyday English in the form of a letter. Some of the follow-up tasks will be useful for enhancing students' written mathematical communication (e.g. Tasks 5 and 6), and the final assessment task (Task 7) is an opportunity for students to demonstrate how their written mathematical communication has developed.

  Guidelines on how to assess the written mathematical communication of Task 8 are given at the end of this booklet, and can certainly be applied to the Rothko problem (Task 1) if you wish. However, it may be more productive to focus on strengthening students' written mathematical communication first before giving them an assessment, which is why we have not included the assessment guide in this task.

Answer: To make n litres of Sail Away Blue,

1 Add the ratio terms together.

That is, if your ratio of paint A to paint B is $a : b$, calculate $a + b$.

Example: The ratio of white to blue is 2 MB : 5W, so $2 + 5 = 7$

2 Calculate each share using the following formulae:

$n \dfrac{a}{(a+b)}$ = the amount of paint A needed

$n \dfrac{b}{(a+b)}$ = the amount of paint B needed

Example:

$4 \times \dfrac{2}{7} = 1.143$ (3 d.p.)

$4 \times \dfrac{5}{7} = 2.857$ (3 d.p.)

So you need 1.143 litres of Moody Blue and 2.857 litres of White paint to make 4 litres of Sail Away Blue paint.