The Essentials of Numeracy

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Abstract

This paper looks at various definitions of numeracy, ranging from dictionary definitions to a literature search. General patterns of these definitions are explored and common themes identified. The place of numeracy in the New Zealand curriculum is outlined both in the Curriculum Framework, where it is described as an essential skill, and in the individual Curriculum Statements. A brief discussion of numeracy and "street-wise maths" is given.

1 Definitions of Numeracy

1.1 Dictionaries and thesauruses

The term numeracy is a relatively new one. Before giving a very brief account of how the term has come to be used in its more technical sense, I will take you through a short journey though some dictionary definitions.

Firstly I tried to find the word in the thesauruses in Word and in Wordperfect. In Word neither *numeracy* nor *numerate* was found. In Wordperfect I did find *numerate* but not *numeracy*. *Numerate* was, however, linked to *enumerate* which in turn links *to list, cite, count* and *number*. At least in this last synonym is a glimmer of hope. Both these thesauruses admitted to the word literacy. In both cases this was not linked to the implications that we usually associate with it, namely reading and writing. The synonyms used were *educated, erudite, learned, lettered, scholarly, schooled, well-read, knowledgeable, cultured.* All of these are high pinnacles of learning. Given that numeracy is often seen as the dual of literacy, the term should imply high levels of cognitive response rather that mere number and computation.

As a second step I tried some dictionary definitions of numerate and literate (numeracy and literacy being derivatives these). All dictionaries had *literate.* These all gave "to read and write". Two mentioned the higher level concept of *being educated* or *having extensive learning or culture*. Interestingly, one mentioned "used to words rather than numbers as a means of expression". This leads to the fascinating concept of creative expression via mathematics. Only some of the dictionaries had *numerate.* This again had multiple definitions: "The ability to use numbers, especially in arithmetic operations", "To read (a numerical expression)", "To enumerate (count)" and two gave the more global "aquainted with the basic principles of mathematics (and science)". Again we see a hierarchy of levels of sophistication. All the dictionaries mentioned the link with literacy, clearly underlying the close relationship between the numeracy and literacy.

Some educational texts define literacy at a low level. For example Good (1973) states that literacy is "(1) strictly the bare ability to read and write; (2) more broadly, the ability to read and write at the level of the average fourth grade pupil", hardly a broad definition. Others like Collins and Downes (1973) are a little more expansive saying it is "The quality of being able

to communicate by means of the written word, or more simply the ability to read and write. This is a matter of degree, hence the term semi-literate". This is starting to suggest some hierarchy of literacy (and analogously numeracy). I subscribe to the views encapsulate by the thesauruses that literacy and numeracy have as their pinnacle the notion of being educated, erudite, cultured and learned. This is to say that numeracy is a high level, hierarchical skill.

A final say from the Oxford Dictionary of New Zealand English. I found no mention of the term *numerate,* but I did find *number eight* (wire. Is this merely a pun or is there a modicum of truth here? The definition of *Number eight* is *the standard number eight (4mm thick) smooth fencing wire especially when used for other than fencing purposes.* Let me observe the use of number, namely eight. Let me further observe, it is especially used when putting the product to use in a wide range of applications. Numerate behaviour can then be analogously defined as:

The standard mathematical tools, especially when used for other than mathematical purposes
Let me call this the number eight definition of numerate behaviour.

The phrase "other than mathematical purposes" clearly implies mathematics employed in real-life contexts. And as number eight wire is used at home, at work, on the farm etc., it clearly also implies the everyday location of where numeracy is applied. These two themes of context and location are common to the majority of definitions of numeracy. The only real difference between number eight and numerate is that the former merely implies a low-tech solution, whereas the latter implies either a low or a high-tech application of mathematics. But like its rural cousin, numeracy is grounded in a common sense approach to problem solving.

It is interesting to note that the latin root for literate is *litteratus* which means learned. And who, then are the literati? They are the men of letters, the learned class, the scholars. So let me introduce a new term to go with the relatively new "numerate". The practitioners (at a high level) of numeracy are the numerati.

1.2 Numeracy definitions in the literature

This look at how the dictionaries define numerate (and hence numeracy), or at least define literate, give a good reflection of the way numeracy is defined in the literature.

The initial reference to numeracy was made by Crowther in 1959. The relationship between numeracy and literacy is emphasised as he states that " numeracy should represent the mirror image of literacy." (as quoted in Cockcroft(1982))

At the very low-level "read-and-write" level of numeracy definition are ones such as the description in Brown et al (1998) of the National numeracy strategy in the UK as "a … set of autonomous basic numerical skills, emphasising mental and written calculations and knowledge of number facts such as multiplication tables." Brown et al go on to say that "while there is reference to contexts in the preamble, they get no nearer to real-life and that the contexts refer only to artificial contexts used in text book word problems". This emphasis on computation and early number concepts alone tends to confuse numeracy with the number strand alone, or even more narrowly equates it with numeration. These are of course the vital foundations of numeracy, but these must be built on in rich ways. I note that the current initiatives in numeracy here in New Zealand, The Early Numeracy Project (ENP) and the

Advanced Numeracy Project (ANP) both heavily emphasise the fundamental place of number. I trust that their plan to branch out into the other stands of mathematics and numeracy comes to fruition.

Most definitions, fortunately, take a much wider and more holistic look at numeracy.

The PISA study define the term "mathematical literacy" (instead of numeracy) as:

> *The capacity to identify, to understand, and to engage in mathematics and to make well-founded judgements about the role that mathematics plays, as needed for the individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen.*

Fancy (2001) for the New Zealand Ministry of Education released this definition:

> *To be numerate is to have the ability and inclination to use mathematics effectively – at home, at work, and in the community.*

Gal (1995) states:

> *The term numeracy describes the aggregation of skills, knowledge, beliefs, dispositions, and habits of mind as well as general communicative and problem solving skills that people need in order to effectively handle real-world situations or interpretive tasks with embedded mathematical or quantifiable elements.*

Willis (1998) says that:

> *Numerate behavior demands a considerable element of what is colloquially called "nous. Some of this nous is mathematical, some is situational (or contextual), and some is strategic.*

Paulos (1988) inverts the definition by defining

> *Innumeracy (is) an inability to deal comfortably with the fundamental notions of number and chance.*

Cockcroft (1982) was an early champion of numeracy. He stated that

> *"numerate" should imply the possession of two attributes. The first is an 'at-homeness' with all those facets of mathematics that enable a person to cope with the practical demands of everyday life. The second is the ability to understand information to understand information presented in mathematical terms. Taken together, these attributes imply that a numerate person should understand some of the ways mathematics can be used for communication.*

1.3 Patterns within the definitions of numeracy

These and a selection of other definitions or statements about numeracy were perused and common themes were looked for (see appendix one)

1.3.1 Location and context

Two common threads within the definitions I shall refer to as *location* and *context*. Location refers to the places in which a person may apply numeracy such as at home, at work, in society etc. The definitions of PISA, and NZMoE are both locational. Context refers to the specific problems, situations, or tasks that the person applies numeracy to. Examples are the definitions of Gal and Willis. Of the forty or so different definitions over two thirds mention one or the other but rarely both.

Table 1: Locational or Contextual definition of Numeracy

	Frequency	**Percentage**
Locational only	17	41.5
Contextual only	11	26.8
Both locational and contextual	4	9.8
Neither locational nor contextual	9	21.8
TOTAL	41	100.0

A key finding was that of the 15 definitions that mentioned context, only one mentioned the cross-curricular, school-based nature of numeracy. This came from Numeracy = Everyone's Business (1997) put out by the Australian Association of Mathematics Teachers. There is an acknowledgement of this aspect by the NZMoE where they mention "… the importance of integration mathematical skills into other curriculum areas". I believe this is vital, as I shall further discuss in the section on numeracy as an essential skill. It is essential not only outside of school, but inside it as well. Many authors should simply add "at school" or "within education" to their lists of locations.

1.3.2 Mathematics in the definition of numeracy

Mathematics is explicitly mentioned in most of the definitions. Sometimes the synonyms "quantitative" or "numerical" are used instead. There are also references to specific disciplines or strands of mathematics (clearly imply the use of mathematics) within some of the definitions. Given this, all of the definitions include mathematics as part of the definition.

Table 2 References to Mathematics in Definitions of Numeracy

	Frequency	Percentage
Mathematics specifically mentioned	24	58.5
Synonyms for mathematics used		
Quantitative	8*	19.5
Numerical Info	2	4.9
Only Strands/Areas of maths used	7	17.0
TOTAL	41	100.0

*One stated “not mathematics but quantitative”.

1.3.3 Mention of the strands of mathematics

On other occasions, people seem to refer to particular disciples (or strands) within mathematics, for example Paulos who mentions number and chance (probability, a subset of the statistics strand). Sometimes the strand was only there by inference, for example “patterns” implies algebra. The results of these can be seen in Table 3.

Table 3 Mention of Strands in Definitions of Numeracy

Strand	Frequency
Number	16
Measurement	6
Geometry	7
Algebra	3
Statistics	10
Mathematical Processes	
Problem Solving	5
Logic and Reasoning	4
Communicating Math, Ideas	5

The totals sum to more than the sample size of 41 as several strands could be mentioned in one definition. Percentages are hence not quoted. Quite a number of definitions made no reference to strands either explicitly of implicitly.

The clear communalities of these trends are for real-life locations and contexts for mathematics. Of the strands, Number is dominant, followed by Statistics, with the Mathematical processes having a good representation. This is of interest as many teachers seem keen for Algebra to be perceived as the next most important strand to Number. Authors like Watson(1998) and Paulos (1988) clearly see Statistics as being a vital life-skill especially as we react to the increasing amount of statistical statements in the media and for decision making in the face of the uncertainty and variability of real-life.

Several authors valued the logical, even the rigorous proof-based approach of mathematics as an element of numeracy. This call for the more formal thinking discipline of mathematics is

encapsulate by Gian Kolata as quoted by Steen (1997) “Quantitative literacy means knowing how to reason and how to think”.

1.4 Numeracy at work and in private lives.

Knight et al have performed a study of the real-life uses of mathematics. This has been done for everyday life and in the workplace. The fifteen everyday tasks ranged from supermarket shopping, to gardening, and to mortgages. The eleven workplace tasks range from counting, to optimising, through to calculator use. I have related each task to the strand(s) of mathematics each involve.

Table 4 Frequency of use Stands of Mathematics in Tasks

	Strands				
Tasks	Number	Measurement	Geometry	Algebra	Statistics
Everyday	12	8	5	1	2
Workforce	4	3	0	1	3
TOTAL	16	11	5	2	5

Again it is clear to see that Number is dominant, with Measurement next. However, Statistics is increasingly useful in the workplace. I believe that the use of statistics as a reflective individual and citizen is a vital skill. I hence elevate Statistics to being alongside Number in its importance as a strands of mathematics. Geometry seems to be used more in everyday life.

Sue Willis (1998) gives an example of the importance of the specific context within workplace based numeracy. She gives an example of a person who is numerate as a chef but not necessarily in other occupations. I should sound one word of warning here. I believe that in the workplace (or even at school) a person can be “pseudo-numerate”, that is appearing to be numerate without being numerate at all. My example came when I employed a bricklayer to build a retention wall. He knew all the rules of thumb on how much concrete and cement to order, and how many barrow loads of concrete this would make. Indeed he was numerate as far as that he could add these amounts up sensibly. But when it came to putting in a small additive to the concrete in a prescribed proportion, he could not apply the principles of proportionality to works out how much per concrete-mixer load he should add. I had to work out a rule of thumb for him, and then he could apply it. Numeracy is not blindly learning and applying rules of thumb, but is showing at least some ability to work out those rules for yourself.

2 Numeracy in the New Zealand Curriculum

2.1 Numeracy as an essential skill

The New Zealand Curriculum Framework defines seven essential learning areas and eight essential skills. The essential skills are intended to be taught across the seven essential learning areas, One of these essential skills is numeracy. The essential numeracy skills are enumerated in the NZCF. They specify that a student will:

1. calculate accurately;
2. estimate proficiently and with confidence;
3. use calculators and a range of measuring instruments confidently and competently;
4. recognise, understand, analyse, and respond to information which is presented in mathematical ways, for example, in graphs, tables, charts, or percentages;
5. organise information to support logic and reasoning;
6. recognise and use numerical patterns and relationships.

Numeration is not included as one of the six essential skills. It is necessary to understand the basics of numeration before numeracy as an essential skill can be meaningfully attained. At the strand level, there is no mention of Geometry.

The New Zealand Council for Educational Research has begun to produce tests of the essential skills. At this time we have completed a suite of standardised tests for Information Skills and are well-progressed on tests of Work and Study Skills. We had begun to develop a suite of tests for numeracy. At this point in time we have put the development on hold as there are a large number of numeracy initiatives currently in full swing. The draft tests have a strong cross-curricular flavour to fit with the concept of the essential skills spanning all the learning areas. As a part of the development we produced our own definition of numeracy which is consistent with the Ministry of Education definition and broadly reflects the main points of numeracy from the earlier analysis.

Our working definition of numeracy was:

> *Numeracy is the ability of a person to make effective use of appropriate mathematical competencies for successful participation in everyday life, including personal life, at school, at work and in the wider community. It involves understanding real-life contexts, applying appropriate mathematical competencies, communicating the results of these to others, and critically evaluating mathematically based statements and results.*

As was seen in earlier in this paper, the relationship between school and numeracy was only once explicitly mentioned. It was far more often directed at out-of-school uses. Numeracy is explicitly an essential skill. For this reason we acknowledged the at-school component in our definition, and then further defined the narrow concept as numeracy as an essential skill. Again it is to be noted that our definition included both the locational and contextual nature of numeracy

We further narrowed the definition of Numeracy as an essential skill.

> *This is a subset of numeracy but has all the hallmarks defined above. At school numeracy should be developed and exhibited across all seven essential learning areas. Out of school numeracy should be exhibited in a range of situations that most students experience in everyday life.*

The following table shows the relationship between the six numeracy skills and the strands of the mathematics curriculum (MiNZC).

Table 5 Areas of mathematics related to the numeracy skills in the NZCF

Numeracy skill	**Mathematics strands involved**	
1. Calculation	**Number**	Computing
	Measurement	Estimation (Money at L2)
	Statistics	Investigations
2. Estimation	**Number**	Computing
	Measurement	Estimation
3. Measurement instruments and calculators	**Number**	Computing
	Measurement	Estimation
		Time, rate and change
4. Information presented in mathematical ways	Number	Exploring number
	Measurement	Time, rate and change
	Statistics	Reporting
		Probability
	Algebra	Patterns
5. Organising information	**Algebra**	Patterns
	Statistics	Investigations
6. Numerical patterns and relationships	Number	Exploring number (L1 to L3)
	Algebra	Patterns

The strands in bold are seen as the ones with the most significant relationship with that essential numeracy skill.

No mention is made of Geometry (either shape or symmetry) or of Algebra – equations.

2.2 Relationship with other curriculum areas

Each of the seven essential learning areas devotes a section to the relationships that learning area has with the essential skills. Each curriculum statement handles this a little differently. The relationship between each of the six numeracy skills and the curriculum area where it is mentioned is shown in table 6

Table 6: Relationships with numeracy mentioned in the curriculum statements

Numeracy skill	Curriculum Statement				
1. Calculation	Science	Technology			
2. Estimation	Science	Technology			
3. Measuring	Science	Technology			Art
4. Math. Info.	Science	Technology	Social Studies	Health	Art
5. Organise info.	Science	Technology	Social Studies	Health	
6. Numerical Patterns	Science				Art

Some aspects numeracy are mentioned in curriculum statements that are not included in the six essential skills - numeracy of the NZCF. Specifically these are:

Science	Using mathematical symbols Using computer spreadsheets (Related to Skill 3 – calculator use) Understanding and expressing uncertainty (Probability or Chance)
Art	Patterns and relationships in …. space (Geometry) Divisions in space (Geometry/Measurement) Repetition and sequence (Geometry – transformations and symmetry/ Algebra – spatial patterns) Variation (Statistics)

These can be addressed by the following strategies:

- Variation could come under Statistics – (either in Investigations or Reporting)
- Probability (or chance) is not explicitly mentioned in the six numeracy skills of the NZCF, but could be included into skill 4, as it is a common mathematical way of presenting information (and is also related to percentage).

Each curriculum statement has a different way of mentioning numeracy.

- English has no mention of numeracy in the curriculum statement. It seems that skills 4, 5 and 6 are all practiced within English. Numeracy skills 4 and 5 are evident in visual language in viewing and presenting functions and processing information. Numeracy skill 6 is evident in the patterns, rhythms and metre of poetic language.
- The mathematics curriculum statement only explicitly mentions skills 1, 2 and 3 in relation to the essential skill of numeracy (see MiNZC p7). Skills 4, 5 and 6 are clearly also present in the mathematics curriculum but their presence is only implied on page 7 of MiNZC. This may be because MiNZC predates the NZCF.
- Science has a detailed page devoted to its relationship with numeracy (see p130 SiNZC). This includes all six numeracy skills, plus some other aspects of numeracy. Other aspects of numeracy are present in the integrating strand entitled *Developing Scientific Skills and Attitudes* (see pp. 42-47 of SiNZC). Essential numeracy skills 3, 4, and 5 are particularly evident there.

- Technology mentions skills 1 to 5 (see p18 TiNZC) with minimal elaboration other than that these skills should be developed through technology. There does seem to be wider issues of numeracy in Technology than this. The strand *Technological Knowledge and Understanding* mentions ideas like "reliability", "efficiency" and "optimisation", as well as investigating the uses of technology, all clearly numeracy/mathematical concepts. The *Technological Capability* strand has the objective of "identifying needs and opportunities", again an area with a numeracy dimension.
- The Social Studies curriculum acknowledges numeracy skills 4 and 5 (see p19 SSiNZC). This is understanding, organising, analysing and responding to numerical and statistical information. Further mention is made in *Social Studies Processes* on pp52-55 under the "inquiry" objective.
- The Health curriculum has a very similar approach to numeracy as does Social Studies with skills 4 and 5 predominating (see p49 HPEiNZC).
- The Arts curriculum has a range of numeracy skills mentioned, in particular skills 2-4 and 6 (see p82 AiNZC). It also includes concepts of space and spatial patterns and strongly hints at symmetry and transformational geometry as numeracy skills.

The place of numerical/statistical information is a little ambiguous. The early curriculum statements such as Mathematics and Science place these in Information Skills, while the latter ones such as Arts, Social Studies and Health placing them more with Numeracy Skills. The NZCF seems to be more consistent with the call of the more recent curricula, and this is also congruent with the authors on numeracy who see statistical literacy a key domain of numeracy.

2.3 Mathematics and Numeracy

Mathematics is a vast body of knowledge and methodologies. It could be defined as "the classification and study of all possible patterns" (Sawyer 1955). Mathematics does not need to look at just the real world, but can look at purely abstract constructs and ideas regardless of their potential applications. Numeracy, on the other hand, is the application of mathematics to real-life situations. The NZCF and MiNZC tend to define mathematics as numeracy (or applied mathematics). MiNZC states "Mathematics is a coherent, consistent and growing body of concepts which make use of specific language and skills to model, analyse, and interpret *the world*." The opening paragraph on mathematics / pangarau from the NZCF states " Everyone needs to learn mathematics. It is essential in most areas of employment. It also is a basic necessity in most other aspects of everyday life. Anyone who wants to keep within a household budget, organise a holiday, prepare for a hui on a marae, wallpaper a room of build a fence needs mathematics". These statements look at a subset of mathematics, commonly called applied mathematics or numeracy. The English mathematician G.H.Hardy has famously been quoted as saying that "No (mathematical) discovery of mine has made, or is likely to make, directly or indirectly, for good or for ill, the least difference to the amenity of the world" (quote in Hoffman(1998)). This emphasises more expansive and abstract nature of at least some mathematics. Ironically, the history of mathematics is one of taking purely abstract mathematics and finding real-life applications for it. Mathematics seems to automatically transmute into numeracy.

The relationship could be thought of as a venn diagram. There is an outer circle of mathematics that is the all-encompassing discipline. Within it is a circle representing numeracy (or applied mathematics in the old jargon). Outside of the numeracy circle is pure

mathematics. Both theses circles intersect with literacy, especially the numeracy circle. Essential skill numeracy is again a subset of the wider notion of numeracy.

This tension between pure and applied mathematics needs to be maintained. I have a strong belief that our current mathematics curriculum is too close to being a numeracy curriculum. While everyone needs to be able to apply mathematics in the real world, mathematics has much to offer as a discipline in its own right. Surely we don't ask "what need do people have for using art, chemistry, history or even fictional writing in their everyday lives?" but we somehow insist on justifying mathematics merely on its everyday uses. Mathematical thinking with its emphasis on logical rigour is a worthy skill, as well as the intrinsic asthetic beauty of the discipline.

Numeracy has a hierarchy of levels ranging from the understanding and use of a few basic ideas of number through to complex mathematical applications. Many of these latter ones relate to highly specific areas of work and hence fall within the numeracy sphere but are excluded from numeracy as an essential skill.

2.4 Numeracy, literacy and information skills

Numeracy is exhibited in real-life situations. These must be communicated through language. This places a literacy load on the student. Numeracy is, however, a distinct domain from literacy. The two domains do of course overlap. Similarly, a student needs to use information skills to obtain relevant inputs to the numeracy process. Numeracy differs from information skills in the way it interprets and uses the information in a distinctively mathematical way.

2.5 Characteristics of numeracy in the essential skills

Tests of numeracy should exhibit a range of the following characteristics:

- common real-life situations are used. Examples of such situations could include balancing a bank account, working out a discount, measuring and mixing quantities, shopping, or reading a newspaper.
- cross-curricula contexts are used i.e. examples drawn from science, technology, social studies, English, art, and health.
- cross-strand uses of mathematics e.g. a mix of number and measurement in the one question.
- competencies of a global nature are exhibited, namely:
    1. choosing to use mathematics;
    2. choosing mathematics appropriate to the context;
    3. correctly applying the mathematics chosen;
    4. interpreting the mathematical findings appropriately for the context;
    5. communicating the findings;
    6. critically evaluating statements of others of a mathematical nature.

Students should use mathematics ably and appropriately. Of these 1, 2, 4 and 6 above are related to how appropriately numeracy is applied, while 3 and 5 relate to how ably the student works. Confidence in their own ability is important to success. This affective domain would need to be tested via a separate instrument.

3 The Numeracy Initiatives and "Street-wise" Numeracy

There is an increasing awareness of the range of problem solving strategies that people bring to mathematical problems. These alternative strategies are being well exploited in the new numeracy initiatives. These alternative strategies are being actively explored and discussed rather than a slavish dependence on just one algorithm. I applaud the richness that this is bringing to mathematics and numeracy, but I would add just a note of warning too. Yes, a diverse range of strategies adds to the numerical nous and number sense of people, that is to say, to their numeracy. However street-wise strategies without robust, generally applicable, and efficient strategies to back them up may leave students somewhat deficient. Two examples will be given that were trialled on a pilot groups of eight students.

Example One – (Piloted on Year 6 students)

Cartons of jam had 32 jam jars per carton.

a) How many jam jars are there in 9 cartons?
b) How many jam jars are there in 24 cartons?
c) Zoe had 544 jam jars. How many cartons would she need to pack them all?

Part a) was answered using by three alternative strategies:

i) Multiplication (1 student).
ii) Successive addition (i.e. 32+32+ …. +32). (3 students).
iii) Using the doubling properties of 32 i.e $2\times32=64$; $64+64=128$ (3 students, 2 of whom were successful).
iv) No clear strategy was used by one student.

Some students showed a street-wise approach to part b).

i) One of the successive adders doubled their answer to a) using addition (i.e. 288 + 288 = 576). They then added six lots of 32 to this total to get 768. They then answered c) by saying "544 is 32 less than 576 so the answer is 17". This person may well struggle with less amenable numbers, given their lack of an efficient algorithm. They are doubtless street-wise.
   The other successive adders failed to correctly add up 24 lots of 32, underlining how an inefficient algorithm breaks down with bigger numbers.
ii) Of the 3 who doubled 32 as a strategy, 1 did not attempt b), and 1got it wrong. The third successfully argued that
   $128 + 128 + 128 = 384$ ($\equiv(4+4+4)\times32\equiv12\times32$); $384 + 384 = 768$ ($\equiv 24\times32$).
   This person then found that $128 + 128 + 128 + 128 +32 = 544$. This is equivalent to four lots of $4\times32 + 32 = 16 \times 32 + 32 = 17 \times 32$.

Example Two - Piloted on Year 10 students

Nick got 33 out of 60 for a project. Write this as a percentage.

i) Two students knew the general algorithm $33/60 \times 100/1$.

ii) One student said "I know 30 / 60 = 50%. The answer is a bit more than this so I guessed 55%" (lucky guess?). Two others guessed close to 55% (52% and 56%). This could have been extended by saying 30 / 60 =50% so 3 / 60 = 5%. Therefore 33 / 60 = 50 % + 5% = 55%.

iii) One student said:

"Two-thirds of 60 = 40. 60 + 40 = 100
Two thirds of 33 = 22 33 + 22 = 55. So its 55%".

These are great examples of streetwise maths (which demonstrate that "at-homeness with numbers" that is a hallmark of numeracy). However none of the street-wise students could proceed onto examples with less accessible numbers which require the more general algorithm. I argue that street-wise maths is great and should be encouraged, but that students should also be exposed to efficient algorithms that work in all situations where their street-wise approaches no longer work or are too prone to error.

4 Conclusion

Numeracy is a relatively newly coined word for the older concept of either "at-homeness with number" or the idea of applied mathematics / social applications.Numeracy is a hierarchical attribute, just as literacy is. It is a worthy goal that all members of society acquire a numeracy level that helps them to fully function at all levels of their lives. This should relate to both the spheres of their lives and the tasks that require mathematical applications that they undertake. Numeracy is an essential skill (as defined in the NZ Curriculum framework). It should be taught and assessed at school in a cross-curricula manner, as numeracy relates to all of the seven essential learning areas. Numeracy should be encouraged as a smart way to approach mathematical problems. Students need also to be given a wide set of mathematical tools to choose from as they solve problems, communicate findings, or react critically to the findings of others in an increasingly quantitative world.

Appendix 1: Some Definitions of Numeracy (Compiled by Karyn Dunn, NZCER)

Crowther's original description, 1959 (UK)

In its report the committee said that 'numeracy' should 'represent the mirror image of literacy'. It should imply 'on the one hand … an understanding of the scientific approach to the study of phenomena …' and 'on the other hand … the need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind'. (Dept of Education and the Arts, 1995,)

Education Department of Western Australia, 1977 (Australia)

The term numerate is understood to mean mathematical literacy … A person is considered to be literate and numerate when he has acquired the skills and concepts which enable him to function effectively in his group and community, and when his attainment in reading, writing, and mathematics makes it possible for him to continue to use these skills to further his own and his community's development.

Mastery of basic number facts (tables)
Competence in operations with whole numbers, fractions, decimals, percentages, money and measurements.
Skills in estimation in relation to these operations, and the habit of making estimates
Sound spatial concepts, including competence with the basic concept of mensuration.
Skill in interpreting graphs
Sound proportion concepts
Statistical literacy based on experiences with the chance processes. (Willis, 1990)

Cockcroft, 1982 (UK)

'Numerate' should imply the possession of two attributes. The first is an 'at-homeness' with all those facets of mathematics that enable a person to cope with the practical demands of everyday life. The second is an ability to understand information presented in mathematical terms - for instance, a graph, chart or table, or a reference to percentage increase or decrease. Taken together, these attributes imply that a numerate person should understand some of the ways mathematics can be used for communication. (Dept of Education and the Arts, 1995)

Saskatchewan Education, 1988 (Canada)

Numeracy can be described as the knowledge, skills and appreciations needed for students to understand and utilize mathematical ideas, techniques and applications. Numerate people demonstrate an "at-homeness" with numbers and are able to understand information that is presented quantitatively: in graphs, charts and tables; through reference to percentage increase or decrease; and in timelines. Numeracy involves students in integrating such skills as interpreting quantitative information, performing straightforward calculations mentally, estimating values that are known and unknown, and developing an intuitive knowledge of measurement units. (Saskatchewan Education, 1998).

Everybody Counts: A Report to the Nation on the Future of Mathematics Education, 1989 (USA)

The authors refer to two kinds of 'literacy': 'verbal' and 'mathematical'. In the context of the report, 'numeracy' is 'mathematical literacy'. The report holds that the two kinds of literacy, '… although different, are not unrelated. Without the ability to read and understand, no one can become mathematically literate. Increasingly, the reverse is also true: without the ability to understand basic mathematical ideas, one cannot fully comprehend modern writing such as that which appears in the daily newspapers.' The report goes on to say that 'Numeracy requires more than just familiarity with numbers. To cope confidently with the demands of today's society, one must be able to grasp the implications of many mathematical concepts - for example, chance. logic and graphs – that permeate daily news and routine decisions.' Citizens must become mathematically literate enough to distinguish evidence from anecdote, recognise nonsense, understand chance, and value proof.
(Dept of Education and the Arts, 1995)

Paulos's 'innumeracy', 1988 (USA)

'Innumeracy [is] an inability to deal comfortably with the fundamental notions of number and chance'.

The Mathematical Needs of NZ School Leavers, 1992 (NZ)

To be numerate is to be able to function effectively mathematically in ones daily life. The research indicates that this includes both of the attributes which the Cockcroft Report described as a feeling for number and feeling for measurement. Specific skills which are required include:

- Both mental and calculator arithemtic skills
- Basic computer skills
- A wide range of measurement skills
- A good understanding of statistical concepts in everyday use
- Simple geometric skills
- Estimation skills
- Problem solving skills including the ability to see alternative ways of handling mathematical problems and to be able to evaluate them

Above all a numerate person will have the confidence ands ability to use these skills in realy life situations as well as in the classroom. (Knight et al, 1992)

A Queensland description, 1994-98 (Australian)

According to the Queensland Department of Education's Literacy and Numeracy Strategy, 'numeracy involves abilities which include interpreting, applying and communicating mathematical information in commonly encountered situations to enable full, critical and effective participation in a wide range of life roles'. It adds that 'numeracy goes well beyond the concept of competence and confidence in computational skills. It incorporates the application of important concepts of number and space in the fields of measurement, approximation, estimation, tables, graphs and statistics.'
(Dept of Education and the Arts, 1995)

A Tasmanian description: Numerate Students – Numerate Adults, 1995 (Australian)

To be numerate is to have and be able to use appropriate mathematical knowledge, understanding, skills, intuition and experience whenever they are needed in everyday life. Numeracy is more than just being able to manipulate numbers. The content of numeracy is derived from five strands of the mathematics curriculum - space, number, measurement, chance and data, and (pattern and) algebra. As students become more numerate, they should become increasingly able to:

- use what they have learned as a basis for further learning;
- add to the variety of situations in which they can use what they have learned;
- recognise when and how mathematics is being used to influence and support arguments;
- consider carefully their own and others' use of mathematics; and
- communicate their mathematical ideas to others. (Dept of Education and the Arts, 1995)

Gal, 1995 (USA/Israel)

The term numeracy describes the aggregate of skills, knowledge, beliefs, dispositions, and habits of mind as well as the general communicative and problem solving skills that people need in order to effectively handle real-world situations or interpretive tasks with embedded mathematical or quantifiable elements.

National Reporting System, 1995 (?)

Using mathematics to make sense of the world; to assist in dealing with real situations which arise in the workplace, personal, and community settings. While it necessarily involves understanding mathematical ideas, notations, and techniques, it also involves drawing on knowledge of particular contexts and circumstances in decided when to use mathematics, choosing the mathematics to use and critically evaluating its use. (Cumming, 1997)

Primary Mathematics Association of South Australia, 1997 (Australian)

Numeracy is the ability to choose and use mathematics to transit a 'real' issue.
(Australian Association of Mathematics Teachers, 1997)

National Benchmarking Taskforce, 1997 (Australian)

Numeracy is the effective use of mathematics to meet the general demands of life at home, in paid work, and for participation in the community and civic life. They will incorporate the development of students understanding and competence with number and quantity (ie measurement), shape and location and the handling and interpretation of quantitative data.
(Australian Association of Mathematics Teachers, 1997)

Numeracy = Everyone's business, 1997 (Australian)

The following elements as central to any definition of numeracy:
numeracy involves using some mathematics to achieve some purpose in a particular context.
To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life. In school education, numeracy is a fundamental component of learning, performance, discourse, and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of:

- underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical, and algebraic)
- mathematical thinking and strategies
- general thinking skills, and
- grounded appreciation of context

Numeracy is both distinct from literacy and complementary to it.
(Australian Association of Mathematics Teachers, 1997)

Askew, Brown, Rhodes, Johnson, and William, 1997 (UK)

Numeracy is the ability to process, communicate, and interpret numerical information in a variety of contexts. (Lokan et al, 2000)

Why Numbers Count: Quantitative Literacy for Tomorrow's America, 1997 (USA)

Quantitative literacy involves understanding the role of numbers in the world. It provides the ability to see below the surface and to demand enough information to get at the real issues.

-- Ted Porter, historian

Beyond arithmetic and geometry, quantitative literacy also requires logic, data analysis, and probability.... It enables individuals to analyze evidence, to read graphs, to understand logical arguments, to detect logical fallacies, to understand evidence, and to evaluate risks. Quantitative literacy means knowing how to reason and how to think.

-- Gina Kolata, journalist

Quantitative literacy can be defined as the level of mathematical knowledge and skills required of all citizens. It includes the ability to apply aspects of mathematics (including measurement, data representation, number sense, variables geometric shapes, spatial visualization, and chance) to understand, predict, and control routine events in people's lives.

-- John Dossey, mathematics educator

Quantitative literacy requires one to understand the nature of mathematics and its role in scientific inquiry and technological progress; to grasp sufficient mathematics to understand important scientific and engineering concepts; and to possess quantitative skills sufficient for responding critically to scientific issues in the media and public life.

-- F. James Rutherford, physics educator

The heart of quantitative literacy is real world problem solving--the use of mathematics in everyday life, on the job, and as an intelligent citizen. Problem solving must be both mathematically defensible and useful in the real world.

-- Henry Pollak, applied mathematician

Numeracy is not the same as mathematics. It is an aggregation of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively and autonomously in quantitative situations arising in life and work. -- Iddo Gal, cognitive scientist

Practices are the habitual patterns of actions engaged in routinely by people, usually without thought; they include standard patterns, routines, procedures, processes, and habits. Quantitative practices deal with numbers, uncertainty, errors in data, design of experiments, creation of models, validations, inferences, making tradeoffs, etc.

-- Peter Denning, computer scientist

Quantitative reasoning as an interpretive activity that takes place within a deductively structured framework. It involves a tapestry of meaning provided by a warp of abstract patterns and a weft of context and story line. In quantitative reasoning, context provides meaning. -- George Cobb, statistician

Important quantitative competencies are those that can be used to solve problems people would frequently encounter on the job or in their roles as citizens or parents. Quantitative competencies require identifying and solving problems not in algebra and geometry, but in the five SCANS competency domains such as planning, information, and systems analysis.

-- Arnold Packer, economist

Quantitative literacy involves understanding the mathematical concepts and skills that are necessary for everyday life. It includes computation, interpretation, inquiry, and application of mathematical concepts that are critical for life in the contemporary world.

-- Glenda Price, college provost

Quantitative literacy involves reasoning with numbers (Jim Lewis); reading, interpreting and making simple applications (Carole Lacampagne); understanding operations on rational numbers (Jack Price); constructing and recognizing a sound argument (Keith Devlin); and understanding variability and how to quantify it (Gail Burrill).

-- Mathematicians and mathematics educators

In contrast, mathematical literacy offers a big-picture view of how to work with numbers, relationships, and patterns (Jim Lewis); higher order thinking, including all the goals of the NCTM Standards (Jack Price); mostly qualitative issues, not quantitative ones (Keith Devlin); the language of algebra as well as geometric and spatial experience (Zal Usiskin).

-- Mathematicians and mathematics educators

(Steen, 1997)

Wainer, 1997 (USA)

Numeracy is a persons proficiency in understanding quantitative phenomena that are presented in a tabular way. (Lokan et al, 2000)

National Numeracy Strategy, 1998 (UK)

Focuses on proficiency regarding numeracy as a culturally neutral and value-free set of autonomous basic numerical skills, emphasizing mental and written calculation and knowledge of number facts such as multiplication tables. Although reference is made to a variety of contexts in the preamble, the detailed bullet points which get no nearer real life than make sense of number problems suggesting that context refers only to the artificial contexts used in text book word problems. (Brown et al, 1998)

Mullis, Martin, Beaton, Gonzalez, Kelly, Smith, 1998 (TIMMS)

Mathematics literacy test was designed to provide information about how prepared all the school leavers in each country are to apply their knowledge in mathematics and science to meet the challenges of life beyond school. (Lokan et al, 2000)

Willis, 1998 (Australia)

Numerate behaviour demands a considerable element of what is colloquially called ‘nous’. Some of this nous is mathematical, some is situational (or contextual), and some is strategic.
(Willis, 1998)

deLange, 1999 (PISA)

Mathematical literacy is an individuals ability, in dealing with the world, to identify, to understand, to engage in, and to make well founded judgements about the role that mathematics plays, as needed for that individuals current and future life as a constructive, concerned, and reflective citizen. (Lokan et al, 2000)

Cooper, 1999 (USA)

A definition of mathematical literacy has been provided that is consistent with and suitable for colloquial use in three senses: education in mathematics, familiarity with mathematical language, and acquaintance with the fundamentals of mathematics.
(Cooper, 1999)

Scott, 1999 (Australian)

In order for numeracy to be the outcome, an individual must be able to confidently, effectively, and appropriately apply mathematics within the given context.

Cummings, 2000 (Australia)

The term numeracy will be considered to be synonymous with the goals of good mathematics education and the directions in which our future mathematical education practices should be heading. It will not be treated as having a separate role to play from mathematics education in the education of Australian students and adults. In making this move to consideration of a single term, all of the previous demands for mathematics education - to learn mathematics, to learn in other disciplines, to do work and for personal lives – must be included.
(Cummings, 2000)

Birmingham Numeracy Strategy, ?? (UK)

Numeracy is a proficiency which involves confidence and competence with numbers and measures. It requires an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve number problems in a variety of contexts. Numeracy also demands practical understanding of the ways in which information is gathered by counting and measuring, and is presented in graphs, diagrams, charts and tables.
(Birmingham's Numeracy Centre)

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